MATH 225: Calculus III Name:________________________

Exam I *October 1, 2002* Instructor:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 5 points each and 4 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

Consider the triangle with vertices $A = (0, 1, 1)$, $B = (-3, 5, 1)$, and $C = (1, 4, -1)$. Find the angle in radians at A.

$\cos^{-1}\left(\frac{9}{5\sqrt{14}}\right)$ $\cos^{-1}\left(\frac{3\sqrt{2}}{\sqrt{35}}\right)$ $\cos^{-1}\left(\frac{8\sqrt{2}}{3\sqrt{35}}\right)$ $\cos^{-1}\left(\frac{16}{5\sqrt{21}}\right)$ $\cos^{-1}\left(\frac{5}{7\sqrt{6}}\right)$

Let $\mathbf{a} = \langle -1, 4, 8\rangle$ and $\mathbf{b} = \langle 2, -3, 6\rangle$. Compute $\text{comp}_{\mathbf{a}}\mathbf{b}$, the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

$34/9$ $34/7$ $34/63$ 62 $\sqrt{62}$

Compute the distance from the point $(-1, 0, -2)$ to the plane $x - 2y + 3z = 7$.

$\sqrt{14}$ $\sqrt{14}/7$ $\sqrt{68}/39$ $\sqrt{68}$ 0

Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \langle 2t, -\sin(t), e^t\rangle$ and $\mathbf{r}(0) = \langle 2, -2, 2\rangle$.

$\langle t^2 + 2, \cos(t) - 3, e^t + 1\rangle$ $\langle 2t^2, \cos(t) - 3, e^t + 1\rangle$ $\langle t^2 + 2, -\cos(t) - 1, e^t + 1\rangle$ $\langle 2t + 2, -\sin(t) - 2, e^t + 1\rangle$ $\langle 2t^2, \cos(t) - 3, 2e^t\rangle$

Determine which of the following expressions gives the length of the curve defined by $\mathbf{r}(t) = 2t\mathbf{i} + \cos t\mathbf{j} + 2\sin t\mathbf{k}$ between the points $(0, 1, 0)$ and $(2\pi, -1, 0)$.

$\int_0^\pi \sqrt{4 + \sin^2 t + 4\cos^2 t}\,dt$ $\int_0^\pi \sqrt{4t^2 + \cos^2 t + 4\sin^2 t}\,dt$ $\int_0^{2\pi} \sqrt{4t^2 + \cos^2 t + 4\sin^2 t}\,dt$ $\int_0^{2\pi} \sqrt{4 + \sin^2 t + 4\cos^2 t}\,dt$ $\int_0^\pi (2\mathbf{i} - \sin t\mathbf{j} + 2\cos t\mathbf{k})\,dt$

Compute the limit $\lim_{(x,y)\to(0,0)} \frac{xy}{2x^2+y^2}$.

does not exist 0 $1/3$ $1/2$ 1

Let $f(x, y) = [\sin(x^2) + \cos(y^2)]e^{-x^2}$. Compute $f_{xy}(\sqrt{\pi/2}, \sqrt{\pi/2})$.

$2\pi e^{-\pi/2}$ $-4\pi e^{-\pi/2}$ $-4\sqrt{\pi/2}e^{-\pi/2}$ $\sqrt{\pi/2}e^{-\pi/2}$ 0

Suppose $f(x, y)$ satisfies $f_x(x, y) = -y/(x-y)^2$ and $f_y(x, y) = x/(x-y)^2$. If $x = u^3 - v$ and $y = uv - v^2$, compute $\partial f/\partial u$ at the point $(u, v) = (1, -1)$.

$1/4$ $-1/8$ $1/2$ -1 0

9. Find a *unit* vector that is perpendicular to both of the vectors $\mathbf{a} = \langle -1, 2, 1 \rangle$ and $\mathbf{b} = \langle 4, -1, 3 \rangle$.

10. Find the point where the line defined by $\mathbf{r}(t) = \langle 2 + t, t, 3 - 3t \rangle$ intersects the plane $3x - y + 2z = 4$.

11. Let C be the curve defined by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ and let M be the plane $x - 3y + 3z = 0$.

(a) Find the points (if any) on C where the *normal plane* to C is parallel to M.

(b) Find the points (if any) on C where the *tangent line* to C is parallel to M.

12. Find the derivative of $f(x,y,z) = xy^2 - x^2z + z^3$ at the point $(3,3,1)$ in the direction parallel to the line $x = 1 + 2t$, $y = -t$, $z = 3 + 2t$.