MATH 225: Calculus III Name:____________________

Exam II *November 12, 2002* Instructor:____________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 5 points each and 4 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems *you must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

$x^2 + 2y^2 + z^2 - xz = 6$ at the point $(0, 1, 2)$.

$x - 2y - 2z = -6$ $x - y - z = -3$ $x - 2y = -2$ $2x - y - z = -3$ $x + 2y + 2z = 0$

Determine which of the following statements is true about the function $f(x, y) = xy + x^4/4 + y^3/3$.

$(0, 0)$ is a saddle point and $(-1, 1)$ is a local minimum. $(0, 0)$ is a local maximum and $(-1, 1)$ is a local minimum. $(0, 0)$ is a saddle point and $(-1, 1)$ is a local maximum. $(0, 0)$ and $(-1, 1)$ are both saddle points. $(0, 0)$ and $(-1, 1)$ are both local maxima.

Evaluate $\int_0^1 \int_x^1 e^{y^2} dy\, dx$ by first reversing the order of integration.

$(e - 1)/2$ $e - 1$ $e/2$ e $2 - e/2$

Let E be the solid region below the inverted cone $z = 1 - \sqrt{x^2 + y^2}$ and above the xy-plane. If the density is uniform, $\rho(x, y, z) = 1$, and the total mass of E is $\pi/3$, find the z-coordinate of the center of mass of E.

$1/4$ $1/3$ $1/2$ $3/8$ $2/9$

Let D be the region in the plane bounded by the curves $y = x$ and $y = x^2$. Evaluate $\int\int_D x^2 dA$.

$1/20$ $1/12$ $1/6$ $1/28$ $1/14$

Find the area of the part of the surface $z = 1 + x$ lying above the triangle in the xy-plane with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$.

$\sqrt{2}/2$ $\sqrt{2}$ $\sqrt{3}$ $\sqrt{3}/3$ $1/4$

Let E be the solid region in the first octant bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + z = 2$, and $y + z = 2$, and lying below the plane $z = 1$. Find $\iiint_E y\, dV$.

$15/8$ $7/4$ $11/6$ 2 $3/2$

Determine which of the following integrals gives the mass of the solid region between the spheres $x^2 +$

$y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 2z$ above the xy-plane, if the density is given by the distance to the z-axis.

$\int_0^{2\pi} \int_0^{\pi/2} \int_{2\cos\phi}^{2} \rho^3 \sin^2\phi \, d\rho \, d\phi \, d\theta$ $\int_0^{2\pi} \int_0^{\pi} \int_0^{2\cos\phi} \rho^3 \sin\phi \, \cos\phi \, d\rho \, d\phi \, d\theta$ $\int_0^{2\pi} \int_0^{\pi/2} \int_2^{2\sin\phi} \rho^3 \sin^2\phi \, d\rho \, d\phi \, d\theta$ $\int_0^{2\pi} \int_0^{\pi} \int_{2\sin\phi}^{4} \rho^2$ si
$\int_0^{2\pi} \int_0^{\pi/2} \int_0^{4} \rho^3 \sin^2\phi \, \cos\phi \, d\rho \, d\phi \, d\theta$

9. Find the absolute maximum and minimum values of the function $f(x, y) = 4x - 4x^2 + y^2$ on the closed triangular region with vertices $(0, 0), (1, 0)$, and $(0, 1)$.

10. Use Lagrange multipliers to find the absolute maximum and minimum of $f(x, y, z) = 4x - 2z$ subject to the constraint $x^2 + y^2 + z^2 = 16$.

11. Find the area of the region *inside* the cardioid $r = 1 + \sin\theta$ and *outside* the cardioid $r = 1 - \sin\theta$.

12. Let R be the region inside the square with vertices $(\pm 1, 0)$ and $(0, \pm 1)$. Evaluate $\iint_R \sqrt{1+y-x}\,\cos(y+x)\,dA$ by making an appropriate change of variables.
[Hint: Start with $u = y + x$ and $v = y - x$.]