MATH 225: Calculus III Name:____________________________

Final Exam *December 17, 2002* Instructor:____________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 20 multiple choice questions worth 6 points each. You start with 30 points.

You may use a calculator if you wish.

Let θ be the angle between the vectors $\mathbf{a} = \langle -2, 4, 1\rangle$ and $\mathbf{b} = \langle -1, 2, -1\rangle$. Find $\cos\theta$.
$\frac{3}{\sqrt{14}}$ $\frac{9}{\sqrt{6}}$ $\frac{9}{\sqrt{21}}$ $\frac{1}{14}$ $\frac{\sqrt{2}}{\sqrt{7}}$

The motion of a particle is described by $\mathbf{r}(t) = \langle 2t^3 - 2t, 3 - 2t^2, 3t - \ln t\rangle$, $t \geq 0$. Find the unit tangent vector at time $t = 1$.
$\frac{1}{3}\langle 2, -2, 1\rangle$ $\frac{1}{\sqrt{5}}\langle 0, 1, 2\rangle$ $\frac{1}{3}\langle 2, 2, 1\rangle$ $\frac{1}{41}\langle 4, 4, 3\rangle$ $\frac{1}{\sqrt{5}}\langle -1, 2, 0\rangle$

Suppose z is defined implicitly as a function of x and y by $xyz = \ln(x + y + z)$. Find $\frac{\partial z}{\partial x}$.
$\frac{1-yz(x+y+z)}{xy(x+y+z)-1}$ $\frac{yz}{1-xy(x+y+z)}$ $\frac{1}{xy(x+y+z)}$ $\frac{1+xy}{x+y+z+1}$ $\frac{yz(x+y+z)}{xy(x+y+z)+1}$

Determine which of the following statements applies to the function
$f(x, y) = x^3 - 3x^2 + 3y^2$.
$(0, 0)$ is *not* a critical point of f f has a saddle point at $(2, 0)$ f has a local maximum at $(0, 0)$ f has a local minimum at $(2, 0)$ *none of the above*

Find the maximum value of the function $f(x, y) = x - y$ on the ellipse $4x^2 + y^2 = 1$.
$\frac{\sqrt{5}}{2}$ $\frac{3\sqrt{3}}{4}$ $\sqrt{2}$ $\frac{1}{2}$ 1

Evaluate $\int_0^1 \int_{x^2}^1 x^5 e^{y^4}\, dy\, dx$.
$\frac{e-1}{24}$ $\frac{e}{4} - \frac{1}{4e}$ $\frac{e}{12}$ $\frac{e}{12} - 1$ $\frac{e-1}{4}$

Let E be the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 4)$. Determine which of the following integrals gives $\iiint_E f(x, y, z)\, dV$.
$\int_0^1 \int_0^{2-2x} \int_0^{4-2y-4x} f(x, y, z)\, dz\, dy\, dx$ $\int_0^1 \int_0^2 \int_0^4 f(x, y, z)\, dx\, dy\, dz$ $\int_0^1 \int_0^{2-x} \int_0^{x+2y} f(x, y, z)\, dz\, dy\, dx$ $\int_0^2 \int_0^{4-2y} \int_0^{4x+2y} f(x, y, z)\, dz\, dx\, dy$ $\int_0^2 \int_0^{1-2y} \int_0^{4-4x-2y} f(x, y, z)\, dz\, dx\, dy$

Find the average of the function $f(x, y, z) = z$ in the region under the unit hemisphere sphere, $0 \leq z \leq \sqrt{1 - x^2 - y^2}$.
3/8 3/16 1/2 1/4 5/16

Let $f(x, y) = x^3 + xy^2$ and suppose x and y are functions of u and v. If $\frac{\partial x}{\partial u} = 3$, $\frac{\partial x}{\partial v} = -2$, $\frac{\partial y}{\partial u} = -4$ and $\frac{\partial y}{\partial v} = 7$ when $(x, y) = (1, 2)$, compute $\frac{\partial f}{\partial u}$ at that point.
5 11 -30 14 49

Find the mass of a thin wire in the shape of the parabola $x = 1 - y^2$, $0 \leq y \leq \sqrt{2}$, if the density is given by $\delta(x, y) = y$.
13/6 $\frac{1}{3}(5^{3/2} - 1)$ $\frac{1}{12}(2^{3/4} - 1)$ 1 11/12

Let $\mathcal{C}$ be the curve $\mathbf{r}(t) = \langle t - 1, t^2 + 1, \frac{1}{t}\rangle$, $1 \leq t \leq 2$. Compute $\int_C (x^2 - y + yz)dy$.
$-\frac{8}{3}$ $\frac{4}{3}$ $-\frac{3}{4}$ 2 0

Find the equation of the plane tangent to the surface parameterized by $\mathbf{r}(u,v) = \langle u^2, 2uv, v^3 \rangle$ at the point $(9,6,1)$.

$x-3y+6z=-3$ $9x+6y+z=0$ $6x+2y+3z=-69$ $6x-18y+z=-49$ $x+y-3z=12$

Evaluate the surface integral $\iint_S z^2\, dS$ where S is the part of the cone $z=\sqrt{x^2+y^2}$ between $z=1$ and $z=2$.

$15\pi\sqrt{2}/2$ $8\pi\sqrt{3}$ $13\pi\sqrt{3}/3$ $7\pi\sqrt{2}/2$ $14\pi/3$

Compute the flux integral $\iint_S \mathbf{F}\cdot\mathbf{n}\, dS$ where $\mathbf{F}(x,y,z) = \langle y^2, -xy, z\rangle$ and S is the paraboloid $z=x^2+y^2$ below $z=1$ with upward orientation.

$\pi/2$ π $-\pi/3$ $\pi/4$ $-2\pi/3$

Calculate $\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = \langle 2xz-y, 2y-x+z, x^2+y+2z\rangle$, and $\mathcal{C}$ is the curve $\mathbf{r}(t) = \langle t, t^2, e^{t-t^2}\rangle$ for $0\le t\le 1$.

2 0 3 -1 1

Use Green's Theorem to calculate the area of the region in the plane enclosed by the curve $\mathbf{r}(t) = \langle \cos t, \sin^3 t\rangle$, $0\le t\le 2\pi$.

$3\pi/4$ $\pi/2$ π 1 2

Find $\operatorname{curl}\mathbf{F}(0,0,0)$ where $\mathbf{F}(x,y,z) = \langle e^x \sin y, e^x\cos y, x+z\rangle$.

$\langle 0,-1,0\rangle$ $\langle 0,0,0\rangle$ $\langle 1,0,0\rangle$ $\langle 0,0,-1\rangle$ $\langle 1,1,1\rangle$

Let $\mathcal{C}$ be the intersection of the cylinder $(x-1/2)^2+y^2=1/4$ and the plane $x+y+z=1$, oriented counterclockwise when viewed from above, and let $\mathbf{F}(x,y,z) = \langle z-y, y, x\rangle$. Calculate $\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$.

$\pi/4$ $\pi/2$ π 2π 0

Let S be the surface $z = 1 - x^2 - y^2$ above the xy-plane with upward orientation, and let $\mathbf{F}(x, y, z)$ be a vector field with $\operatorname{curl} \mathbf{F}(x, y, z) = \langle y - 4xz, 2yz - x, z^2 \rangle$. Compute $\iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{n} \, dS$.

0 π $\pi/4$ $\pi/2$ 2π

Let S be the surface of the cube with vertices $(\pm 1, \pm 1, \pm 1)$ with outward orientation. Calculate the flux of the vector field $\mathbf{F}(x, y, z) = \langle e^y, e^x, z \rangle$ across S.

8 4 2 0 16