MATH 225: Calculus III **Practice Exam II** *November 2002*

Suppose the plane $2x - y + z = 0$ is tangent to the level surface $f(x, y, z) = 2$ at the point $(0, 1, 1)$. Determine which of the vectors below is parallel to $\nabla f(0, 1, 1)$.

$\langle -2, 1, -1\rangle$ $\langle 1, 2, -1\rangle$ $\langle 2, -1, -1\rangle$ $\langle 4, -2, -1\rangle$ $\langle 4, 2, -2\rangle$

Let $f(x, y) = 8x^3 + 6xy + y^3$. Determine which of the following statements is true.

$(0, 0)$ is a saddle point and $(-1/2, -1)$ is a local maximum. $(0, 0)$ is a saddle point and $(-1/2, -1)$ is a local minimum. $(0, 0)$ is a local maximum and $(-1/2, -1)$ is a local minimum. $(0, 0)$ is a local minimum and $(-1/2, -1)$ is a local maximum. $(0, 0)$ is a local maximum and $(-1/2, -1)$ is a saddle point.

Let C be the curve of intersection of the surfaces $x^2 + y^2 + z^2 = 1$ and $xy + z^2 = 0$. Determine which of the following systems of equations must be solved to find the point(s) on C closest to $(1, 1, 1)$ using Lagrange multipliers.

$$\begin{aligned}
2(x-1) &= 2\lambda x + \mu y\\
2(y-1) &= 2\lambda y + \mu x\\
(z-1) &= \lambda z + \mu z\\
x^2 + y^2 + z^2 &= 1\\
xy + z^2 &= 0
\end{aligned}$$

$$\begin{aligned}
2x &= 2\lambda(x-1) + \mu y\\
2y &= 2\lambda(y-1) + \mu x\\
2z &= \lambda(z-1) + \mu z\\
x^2 + y^2 + z^2 &= 1\\
xy + z^2 &= 0
\end{aligned}$$

$$\begin{aligned}
(x-1) &= \lambda x\\
(y-1) &= \lambda y\\
(z-1) &= \lambda z\\
x^2 + y^2 - xy &= 1
\end{aligned}$$

$$\begin{aligned}
2(x-1) &= \lambda y\\
2(y-1) &= \lambda x\\
(z-1) &= \lambda z\\
x^2 + y^2 - xy &= 1
\end{aligned}$$

$$2(x-1) = \lambda(2x-y)$$
$$2(y-1) = \lambda(2y-x)$$
$$2(z-1) = \lambda(x+y)$$
$$x^2+y^2-xy = 1$$

Reverse the order of integration in the double integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$.

$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$ $\int_0^{\sqrt{1-x^2}} \int_{-1}^{1} f(x,y)\,dx\,dy$ $\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^1 f(x,y)\,dx\,dy$ $\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,d$

$\int_{-1}^{1}\int_0^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

Evaluate the double integral $\int_0^1\int_0^{x^2} xe^y\,dy\,dx$.

$\frac{1}{2}e-1$ $\frac{1}{2}(e-1)$ $\frac{1}{2}e$ $e-\frac{1}{2}$ $e-1$

Find the area inside the curve $r = 2+\cos(\theta)$. $\frac{9\pi}{2}$ $\frac{3\pi}{2}$ 6π $\frac{15\pi}{2}$ 3π

Determine which of the following double integrals gives the area of the part of the surface $z = x+y^3$ that lies above the region bounded by $x = y^2$, $y = 0$, and $x = 1$.

$\int_0^1\int_{y^2}^1 \sqrt{2+9y^4}\,dx\,dy$ $\int_0^1\int_0^{\sqrt{x}} \sqrt{2+3y^2}\,dy\,dx$ $\int_0^1\int_{\sqrt{x}}^1 \sqrt{2+3y^2}\,dy\,dx$ $\int_0^1\int_0^{y^2} \sqrt{2+9y^4}\,dx\,dy$

$\int_0^1\int_{y^2}^1 \sqrt{1+x^2+9y^4}\,dx\,dy$

Let E be the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,2)$. Express the integral $\iiint_E f(x,y,z)\,dV$ as an iterated integral.

$\int_0^1\int_0^{2-2x}\int_0^{2-2x-y} f(x,y,z)\,dz\,dy\,dx$ $\int_0^1\int_0^{1-y}\int_0^{(2-y-z)/2} f(x,y,z)\,dx\,dz\,dy$ $\int_0^1\int_0^2\int_0^2 f(x,y,z)\,dz\,dy\,dx$

$\int_0^2\int_0^{1-y}\int_0^{2-x-2y} f(x,y,z)\,dz\,dx\,dy$ $\int_0^2\int_0^1\int_0^{2-y} f(x,y,z)\,dz\,dx\,dy$

Let E be the region between the spheres $x^2+y^2+z^2 = z$ and $x^2+y^2+z^2 = 2z$. Which of the following represents $\iiint_E (x^2+y^2)\,dV$ in spherical coordinates.

$\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos(\phi)}^{2\cos(\phi)} \rho^4\sin^3(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos(\phi)}^{2\cos(\phi)} \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi}\int_0^{\pi/2}\int_1^2 \rho^4\sin(\phi)\,d\rho\,d\phi\,d\theta$

$\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos(\theta)}^{2\cos(\theta)} \rho^4\sin^3(\theta)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}\int_{\cos(\phi)}^{2\cos(\phi)} \rho^4\sin^3(\phi)\,d\rho\,d\phi\,d\theta$

Let $\mathcal{C}$ be the curve parameterized by $\mathbf{r}(t) = \langle t, t^2, t^2\rangle$, $0 \le t \le 1$. Evaluate the line integral $\int_{\mathcal{C}} x\,ds$.

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11. Find the absolute maximum and minimum values of $f(x,y) = x^2+y^2+x+1$ on the unit disk $x^2+y^2 \le 1$.

12. Let D be the triangular region in the plane with vertices $(0,0)$, $(2,0)$, and $(1,2)$. Compute $\iint_D x\,dA$.

13. A lamina of uniform density occupies the region D bounded by the parabola $y = 1-x^2$ and the x-axis. Find its center of mass.

14. Let R be the region bounded by the hyperbolas $xy = 1$ and $xy = 9$ and the lines $y = x$ and $y = 4x$ in the first quadrant. Use the change of variables $x = uv$, $y = u/v$, $u \geq 0$, $v \geq 0$, to evaluate the integral $\iint_R x^2 y \, dA$.