MATH 225: Calculus III **Practice Final Exam** *December 2002*

Find the arc length of the curve described by the vector function $\mathbf{r}(t) = 2\cos(\pi t^2)\mathbf{i} + \sqrt{2}\sin(\pi t^2)\mathbf{j} - \sqrt{2}\sin(\pi t^2)\mathbf{k}$ for $0 \le t \le 1$.

2π 4π 2 4 $16\pi^2/3$

Let $\mathbf{v} = \langle t, t^2, t^3\rangle$ for a real number t, and let $\mathbf{w} = \langle 1, 1, 1\rangle$. Find the vector projection $\mathrm{proj}_{\mathbf{w}}(\mathbf{v})$ of $\mathbf{v}$ on $\mathbf{w}$.

$\frac{t+t^2+t^3}{3}\langle 1,1,1\rangle$ $\frac{t^2+t^4+t^6}{3}$ $\frac{t+t^2+t^3}{\sqrt{3}}$ $(t+t^2+t^3)\langle 1,1,1\rangle$ $\frac{t+t^2+t^3}{\sqrt{3}}\langle t,t^2,t^3\rangle$

Suppose that the acceleration of an object at time $t \ge 0$ is given by $\mathbf{a}(t) = 2t\mathbf{i} + 2\mathbf{k}$, that the object's position at time $t = 0$ is $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$, and that its position at time $t = 1$ is $\mathbf{r}(1) = \mathbf{0}$. Find $\mathbf{v}(0)$, the object's velocity at time $t = 0$.

$-4/3\mathbf{i} - \mathbf{j} - \mathbf{k}$ 0 $-2\mathbf{i} - \mathbf{j} - \mathbf{k}$ $t^2\mathbf{i} + 2t\mathbf{k}$ $\mathbf{i} + 2\mathbf{k}$

Determine which of the following represents the area of the parallelogram consisting of the points

$$(u+1, 2u+3v+1, -u+v+1), \quad 0 \le u \le 1, \quad 0 \le v \le 1$$

$|(\mathbf{i}+2\mathbf{j}-\mathbf{k})\times(3\mathbf{j}+\mathbf{k})|$ $|(\mathbf{i}+5\mathbf{j})\times(\mathbf{i}+\mathbf{j}+\mathbf{k})|$ $|(2\mathbf{i}+3\mathbf{j})\times(\mathbf{i}+4\mathbf{j}+2\mathbf{k})|$ $|(\mathbf{i}+2\mathbf{j}-\mathbf{k})\times(\mathbf{i}+4\mathbf{j}+2\mathbf{k})|$ $(\mathbf{i}+2\mathbf{j}-\mathbf{k})\cdot(3\mathbf{j}+\mathbf{k})$

Find equations for the line of intersection of the planes $x - y + z = 2$ and $2x + z = 1$.

$x = -t$, $y = -1+t$, $z = 1+2t$ $x = 1/2-t$, $y = -3/2-t$, $z = t$ $x = t$, $y = -1+t$, $z = 1-2t$ $x = 1/2+t$, $y = -3/2$, $z = -t$ $x = 1/2+t$, $y = -1-t$, $z = -2t$

Compute $\lim_{(x,y)\to(0,0)} \frac{x^2+y^4}{\sqrt{x^2+y^4+4}-2}$.

4 0 1 $-1/2$ *does not exist*

Let z be the function of x and y defined by equation $xz + e^z y = 2$. Find $\frac{\partial z}{\partial y}$ at the point $(1, 2, 0)$.

$-1/3$ -1 0 2 -3

Find the maximum rate of change of the function $f(x,y,z) = x^2y + e^x z^2$ at point $(0, 1, -1)$.

$\sqrt{5}$ 1 $\sqrt{2}$ 0 $\sqrt{3}$

Determine which of the following statements describes the function

$$f(x,y) = x^3 - y^3 - 2xy + 6$$

at point $(-2/3, 2/3)$.

f has a local maximum. f has a local minimum. f has a saddle point. The point is not a critical point. The second derivative test is inconclusive.

Find the minimum of $f(x,y,z) = (x-1)^2 + y^2 + z^2$ subject to the constraint $x^2 - yz = 0$.

$2/3$ $1/3$ $1/2$ 1 2

Evaluate $\int_0^1 \int_{e^y}^e \frac{1}{\ln(x)}\,dx\,dy$. (Hint: Change the order of integration.)

$e-1$ 1 -1 e $1/\ln(2)$

Determine which of the following integrals gives the volume of the solid inside the cylinder $x^2 + y^2 = 2x$ above the xy-plane and below the cone $z = \sqrt{x^2+y^2}$.

$\int_{-\pi/2}^{\pi/2}\int_0^{2\cos(\theta)}\int_0^r r\,dz\,dr\,d\theta$ $\int_0^{2\pi}\int_0^2\int_0^r r\,dz\,dr\,d\theta$ $\int_{-\pi/2}^{\pi/2}\int_0^{2\cos(\theta)}\int_0^1 z\,dz\,dr\,d\theta$ $\int_{-\pi/2}^{\pi/2}\int_0^2\int_0^z r\,dz\,dr\,d\theta$ $\int_0^{\pi}\int_0^{2\cos(\theta)}\int_0^r r\,dz\,dr\,d\theta$

Evaluate the integral $\int_1^4\int_0^{1/\sqrt{x}} e^{\sqrt{x}}\,dy\,dx$ by using the change of variables $x = u^2$, $y = v$. $2(e^2-e)$ $e-1$ $2/e$ $4e^2-e$ $e^{\sqrt{3}}-1$

Let E be the solid bounded by the cylinder $z = 1-x^2$ and the plane $y = z$ in the first octant. Determine which of the following integrals equals $\iiint_E f(x,y,z)\,dV$.

$\int_0^1\int_0^{1-x^2}\int_0^z f(x,y,z)\,dy\,dz\,dx$ $\int_0^1\int_0^{\sqrt{1-z}}\int_0^1 f(x,y,z)\,dz\,dx\,dy$ $\int_0^1\int_z^1\int_0^{\sqrt{1-z}} f(x,y,z)\,dx\,dy\,dz$ $\int_0^1\int_0^1\int_0^{1-x^2} f(x,y,z)\,dz\,dx\,d$ $\int_0^1\int_0^{1-x^2}\int_z^1 f(x,y,z)\,dy\,dz\,dx$

Let E be the solid between the concentric hemispheres $x^2+y^2+z^2 = 16$, $z \ge 0$, and $x^2+y^2+z^2 = 4$, $z \ge 0$. Assuming the density at each point is inversely proportional to its distance from the origin, find the mass of E. (Let k denote the constant of proportionality.)

$12k\pi$ $2k\pi\ln(2)$ $48k\pi$ $8k\pi$ $8k\pi^2$

Let $\mathcal{C}$ be the curve $\mathbf{r}(t) = \langle \sin(3t), \sin(t) + 2\sin(2t), \cos(t) - 2\cos(2t)\rangle$, $0 \le t \le 2\pi$. Compute $\int_{\mathcal{C}} y\,dx + x\,dy + z\,dz$.

0 1/2 1 2 3/2

Find the equation of the plane tangent to the surface parameterized by

$$\mathbf{r}(u, v) = \langle u - v^2, v - u^2, uv\rangle, \quad -2 \le u \le 2, \quad -2 \le v \le 2$$

at the point $(0, 0, 1)$.

$x + y + z = 1$ $y + z = 1$ $x - y + z = 0$ $x + y - z = 0$ $-x - y + z = 1$

Suppose the temperature at a point (x, y, z) on the hemisphere $z = \sqrt{1 - x^2 - y^2}$ is $T(x, y, z) = 20z^2$. Determine the average temperature on the hemisphere. The area of the surface is 2π.

6.67 10 2.09 15 12.33

Let $\mathcal{C}$ be the intersection of the cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 4$ with $z \ge 0$, oriented counterclockwise when viewed from above. Evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = y\mathbf{i} + 2x\mathbf{j} + e^z\mathbf{k}$.

π 2π 0 $\sqrt{2} + \pi$ $\sqrt{2}/2 + 2\pi$

Let S be the sphere $x^2 + y^2 + z^2 = 1$ and let $\mathbf{n}$ be the outward unit normal vector to S. Calculate the flux integral $\iint_S \mathbf{F} \cdot \mathbf{n}\,dS$ where $\mathbf{F}(x, y, z) = xy^2\mathbf{i} + yz^2\mathbf{j} + zx^2\mathbf{k}$.

$4\pi/5$ $2\pi/3$ $\pi/2$ 0 π

Let $\mathbf{F}(x, y) = (x+y)\mathbf{i} + (y-x)\mathbf{j}$. Calculate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathcal{C}$ is the circle $\mathbf{r}(t) = \langle \cos(t), \sin(t)\rangle$, $0 \le t \le 2\pi$.

-2π 0 $3\pi/2$ -1 π

Let $\mathbf{F}$ be a vector field defined on $\mathbf{R}^3$. Determine which of the following conditions guarantees that $\mathbf{F} = \nabla f$ for some function f.

$\operatorname{curl}\mathbf{F} = \mathbf{0}$ $\operatorname{div}\mathbf{F} = 0$ $\operatorname{div}\operatorname{curl}\mathbf{F} = 0$ $\operatorname{curl}\operatorname{curl}\mathbf{F} = \mathbf{0}$ $\operatorname{grad}\operatorname{div}\mathbf{F} = \mathbf{0}$

Let $\mathbf{F} = (e^x + yz)\mathbf{i} + (e^y - xz)\mathbf{j} + (\sin(z) + x - y)\mathbf{k}$. Compute $\operatorname{div}\mathbf{F}$.

$e^x + e^y + \cos(z)$ $e^x\mathbf{i} + e^y\mathbf{j} + \cos z\mathbf{k}$ $z - x + 1$ $y + z + \cos z$ $(x-1)\mathbf{i} + (y-1)\mathbf{j} + 2z\mathbf{k}$

Determine which of the following integrals gives the area of the surface parameterized by $\mathbf{r}(u, v) = u^2\mathbf{i} + uv\mathbf{j} + v^2\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le 2$.

$\int_0^2\int_0^1 2\sqrt{u^4 + 4u^2v^2 + v^4}\,du\,dv$ $\int_0^2\int_0^1 \sqrt{u^4 + u^2v^2 + v^4}\,du\,dv$ $\int_0^2\int_0^1 u^4 + u^2v^2 + v^4\,du\,dv$ $\int_0^2\int_0^1 \sqrt{2}(u^2 + 4uv + v^2)\,du\,dv$ $\int_0^2\int_0^1 2(u^2 + v^2)\,du\,dv$

Determine which of the following plots is the surface parameterized by $\mathbf{r}(u, v) = u\cos(v)\mathbf{i} + u\sin(v)\mathbf{j} + v\mathbf{k}$, $-1 \le u \le 1$, $0 \le v \le 2\pi$.

0.1in

26. Let $\mathbf{F} = (e^y + yz)\mathbf{i} + x(e^y + z)\mathbf{j} + (xy - 2)\mathbf{k}$.

a) Find a function f such that $\mathbf{F} = \nabla f$.

b) Use the Fundamental Theorem for Line Integrals to evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathcal{C}$ is a curve from $(0, 0, 0)$ to $(3, 1, -1)$.

0.1in

27. Use Green's Theorem to calculate

$$\int_C (e^{x^2} + xy)dx + (x + \sin(y^2))dy$$

where C is the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$, oriented counter-clockwise.