MATH 225: Calculus III Name:________________________

Exam I *September 30, 2003* Instructor:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 5 points each and 4 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

Let $\mathbf{a} = \langle 1, -1, 1\rangle$ and $\mathbf{b} = \langle -1, 1, 1\rangle$. Compute $\text{proj}_{\mathbf{a}}\mathbf{b}$, the vector projection of $\mathbf{b}$ onto $\mathbf{a}$.
$\langle -1/3, 1/3, -1/3\rangle$
$\langle -1/3, 1/3, 1/3\rangle$
$\langle -1, 1, -1\rangle$
$\langle 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\rangle$
$\langle 2, 1, 0\rangle$

Determine which of the following is true for all unit vectors $\mathbf{a}$ and $\mathbf{b}$ with the property that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.
$(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b}) = 2.$
$(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = 2.$
$(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b}) = 4.$
$(\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = -1.$
$(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = 1.$

Find the distance from the point $(1, 0, 3)$ to the plane $2x - y - 2z = -1$.
1
$1/\sqrt{6}$
$4/3$
$5/3$
$1/\sqrt{3}$

Find the direction in which $f(x, y, z) = z^2 e^{y(1-x)}$ has the maximum rate of change at the point $(1, 1, 1)$.
$\langle -1, 0, 2\rangle$
$\langle 1, 1, 1\rangle$
$\langle 1, 1, 2\rangle$
$\langle -1, 0, 1\rangle$
$\langle -1, 1, 2\rangle$

Find all the values of t, if any, where the curve parameterized by

$$\mathbf{r}(t) = \langle te^{-t^2/2}, t^3 - 3t, \cos(\pi t)\rangle$$

is *not* smooth.
$t = -1, 1$
$t = 1$
$t =$ any integer
$t = 0, 1/2, -1/2, \sqrt{3}, -\sqrt{3}$
No values of t.

Find the equation of the osculating plane (i.e., the plane determined by $\mathbf{T}$ and $\mathbf{N}$) for the curve parameterized by $\mathbf{r}(t) = \langle t^2 - t, t^3, t^2 + t\rangle$ at the point $(2, 8, 6)$.
$9x - y - 3z = -8$
$3x + 12y + 5z = 132$
$x + 3y + 3z = 44$

$3x - y = -2$
$31x - 54y + 111z = 296$
Determine the limit $\lim_{(x,y)\to(0,0)} \frac{\sqrt{4+x^2+y^2}-2}{x^2+y^2}$.
$\frac{1}{4}$
$\frac{1}{2}$
1
0
does not exist
Let $f(x,y) = x\sin(x+y)$. Calculate $f_{xx} + f_{xy} + f_{yx}$.
$4\cos(x+y) - 3x\sin(x+y)$;
$2\cos(x+y) + 3x\sin(x+y)$;
$4\cos(x+y) - 2x\sin(x+y)$;
$2\cos(x+y) - 2x\sin(x+y)$;
$2\sin(x+y) - 3x\cos(x+y)$.

9. Find a unit vector in the first octant and in the plane $z = x$ that makes a 45° angle with the y-axis.

10. Find the parametric equations of the line of intersection of the planes $x - y + 2z = 2$ and $x + 2y + z = -1$.

11. Find the domain and range of the function $f(x, y) = \frac{4}{1+x^2+y^2}$. Then sketch several level curves of f and use them to sketch the graph of f. Be sure to show all your work and compare the level curves to the graph.

12. Suppose $f(x, y, z) = x^2y + y^3z + z^4x$ where $x = x(t)$, $y = y(t)$, $z = z(t)$ are the components of a vector function $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ satisfying $\mathbf{r}(1) = \langle 2, 0, 1\rangle$ and $\mathbf{r}'(1) = \langle 3, -4, 6\rangle$. Find $\frac{df}{dt}$ at $t = 1$.