MATH 225: Calculus III Name:____________________________

Exam II *November 11, 2003* Instructor:____________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 5 points each and 4 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

Determine the equation of the line perpendicular to the surface $z = 1 + (x+y)e^y$ at the point $(0,0,1)$.
$x = t,\ y = t,\ z = 1 - t$
$x = 0,\ y = 2t,\ z = 1 + t$
$x = t,\ y = t,\ z = 1$
$x = t,\ y = 2t,\ z = 1 - 2t$
$x = 2t,\ y = t,\ z = 1 + 2t$
Find all the critical points of $f(x,y) = 2y^3 + 3x^2y + 5y^2 + 2x^3$.
$(0,0),\ (0,-\frac{5}{3}),\ (\frac{10}{9},-\frac{10}{9})$
$(0,0)$
$(0,0),\ (0,-\frac{5}{3})$
$(0,0),\ (0,-\frac{5}{3}),\ (-\frac{10}{9},\frac{10}{9}),\ (\frac{10}{9},-\frac{10}{9})$
$(0,0),\ (0,\pm\frac{5}{3}),\ (\pm\frac{10}{9},\pm\frac{10}{9})$
Find the statement that describes the value of the iterated integral $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} 2 - \sqrt{4 - x^2 - y^2}\, dx\, dy$.
The volume of a half of a cylinder with a quarter of a sphere removed.
The volume of a quarter of sphere.
The volume of a cylinder with a half of a sphere removed.
The volume of a half of a cylinder with an eighth of a sphere removed.
The volume of an eighth of sphere.
Reverse the order of integration in the iterated integral $\int_0^2 \int_{y^2-1}^{2y-1} f(x,y)\, dx\, dy$.
$\int_{-1}^3 \int_{(x+1)/2}^{\sqrt{x+1}} f(x,y)\, dy\, dx$
$\int_{y^2-1}^{2y-1} \int_0^2 f(x,y)\, dy\, dx.$
$\int_2^0 \int_{2y-1}^{y^2-1} f(y,x)\, dy\, dx.$
$\int_0^4 \int_{1+x/2}^{\sqrt{x+1}} f(x,y)\, dy\, dx.$
$\int_1^3 \int_{1+\sqrt{x}}^{(x+1)/2} f(x,y)\, dy\, dx$
Let E be the solid in the first octant bounded by the cylinder $y^2 + z^2 = 4$ and the plane $y = 2x$. Find the iterated integral that gives $\iiint_E f(x,y,z)\, dV$.

$\int_0^1 \int_{2x}^2 \int_0^{\sqrt{4-y^2}} f(x,y,z)\, dz\, dy\, dx$
$\int_0^2 \int_0^{y/2} \int_x^{\sqrt{4-z^2}} f(x,y,z)\, dy\, dx\, dz$
$\int_0^1 \int_0^{2x} \int_0^{\sqrt{4-y^2}} f(x,y,z)\, dz\, dy\, dx$
$\int_0^2 \int_{y/2}^1 \int_0^{\sqrt{4-z^2}} f(x,y,z)\, dy\, dx\, dz$
$\int_0^1 \int_{2x}^2 \int_0^{2\sqrt{1-x^2}} f(x,y,z)\, dz\, dy\, dx$

Determine which iterated integral gives the area of the part of the surface $z = 1 + x^2 + y^2$ that lies above the square $0 \le x \le 1$, $0 \le y \le 1$.

$\int_0^1 \int_0^1 \sqrt{1 + 4x^2 + 4y^2}\, dx\, dy$
$\int_0^1 \int_0^1 \sqrt{1 + 2x + 2y}\, dx\, dy$
$\int_0^1 \int_0^1 \sqrt{1 + x^2 + y^2}\, dx\, dy$
$\int_0^1 \int_0^1 \int_0^{1+x^2+y^2} dz\, dx\, dy$
$\int_0^1 \int_0^1 \int_1^3 2\sqrt{x^2 + y^2}\, dz\, dx\, dy$

Determine which iterated integrals gives the mass of the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $3z^2 = x^2 + y^2$ if the density is $\delta(x, y, z) = x^2 z$.

$\int_0^{2\pi} \int_0^{\pi/3} \int_0^2 \rho^5 \sin^3(\phi) \cos(\phi) \cos^2(\theta)\, d\rho\, d\phi\, d\theta$
$\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \rho^3 \sin^2(\phi) \cos(\phi) \cos^2(\theta)\, d\rho\, d\phi\, d\theta$
$\int_0^{2\pi} \int_0^{\pi/3} \int_{\sqrt{3}}^2 \rho^5 \cos^3(\phi) \sin(\phi) \cos^2(\theta)\, d\rho\, d\phi\, d\theta$
$\int_0^{2\pi} \int_0^{\pi/6} \int_{\sqrt{3}}^2 \rho^3 \cos^2(\phi) \sin(\phi) \sin^2(\theta)\, d\rho\, d\phi\, d\theta$
$\int_0^{2\pi} \int_0^{\pi/6} \int_0^2 \rho^5 \sin^2(\phi) \cos(\phi) \cos^2(\theta)\, d\rho\, d\phi\, d\theta$

Calculate the volume of the solid region between the surfaces $z = 6 - x^2 - y^2$ and $z = \sqrt{x^2 + y^2}$ over the domain $1 \le x^2 + y^2 \le 4$.

$35\pi/6$
$13\pi/3$
3π
$8\pi/3$
$71\pi/6$

9. Find the local maximum and minimum values and saddle points, if any, of the function $f(x, y) = x^2 + 2y^2 - x^2y$.

10. Determine the minimum distance to the origin of the points on the curve defined by $x^2 - 7xy + y^2 = 36$.

11. Let D be the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$ in the first quadrant. The area of D is $\pi/2$. Find the average value of $f(x, y) = y$ over D.

12. Compute the double integral $\iint_D (x+y)\, dA$ where D is the region bounded by the ellipse $(x-2)^2+4(y-1)^2 = 16$ by making the change of variables $x = 2 + 4r\cos\theta$, $y = 1 + 2r\sin\theta$.