**MATH 225: Calculus III** Name:_________________________

**Final Exam** *December 17, 2003* Instructor:_________________________

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this page. There are 20 multiple choice questions worth 6 points each. You start with 30 points.

You may use a calculator if you wish.

Find the scalar projection, $\text{comp}_{\mathbf{w}}(\mathbf{v})$, of the vector $\mathbf{v} = \langle 1, 1, 2\rangle$ onto the vector $\mathbf{w} = \langle 2, -2, 1\rangle$.

$2/3$
$2/\sqrt{6}$
$2$
$\sqrt{6}/9$
$1$

Calculate the arc length of the helix parameterized by $\mathbf{r}(t) = \big\langle 4t, 3\cos(t), 3\sin(t)\big\rangle$ for $0 \le t \le 2\pi$.

$10\pi$
$0$
$2\pi$
$6\pi$
$12\pi$

Find a direction vector for the line of intersection of the planes $x + y + 2z = 1$ and $3x - y = 0$.

$\langle 1, 3, -2\rangle$
$\langle 3, 1, -2\rangle$
$\langle 3, -2, 1\rangle$
$\langle -2, 3, 1\rangle$
$\langle -2, 1, 3\rangle$

A particle starts from rest at time $t = 0$ at the origin $(0, 0, 0)$. It then begins to move with acceleration $\mathbf{a}(t) = \big\langle 1, t, t^2\big\rangle$. Find the time, if ever, at which the particle passes through the point $\big(2, \frac{4}{3}, \frac{8}{3}\big)$.

never
$t = 1$
$t = 2$
$t = 3$
$t = 4$

If $z = f(x, y)$, $x = u^2 + v^2$ and $y = u^2 - v^2$, find $\frac{\partial^2 z}{\partial u \partial v}$.

$4uv\big(f_{xx} - f_{yy}\big)$
$4uv\big(f_{xx} + f_{yy}\big)$
$4uv\big(f_{xx} + 2f_{xy} - f_{yy}\big)$
$2u\big(f_{xx} + 2f_{xy} + f_{yy}\big)$
$2\big(f_{xx} + f_{yy}\big)$

Find the maximum rate of change of $f(x, y) = x^2y + 2y$ at the point $(-1, 2)$ and the direction in which it occurs.

The maximum rate of change is 5 in the direction of $-4\mathbf{i} + 3\mathbf{j}$.
The maximum rate of change is 6 in the direction of $4\mathbf{i} - 3\mathbf{j}$.
The maximum rate of change is $-5$ in the direction of $-4\mathbf{i} + 3\mathbf{j}$.
The maximum rate of change is 4 in the direction of $-3\mathbf{i} + 4\mathbf{j}$.
The maximum rate of change is $-4$ in the direction of $3\mathbf{i} - 4\mathbf{j}$.

Find the minimum of the distance to the origin of the points on the surface $xy^2z^2 = 4$.

$\sqrt{5}$
$5$
$1$
$\sqrt{3}$
$\sqrt{7}$
Find the maximum and minimum values of $f(x,y) = 2x^2 + 4x + y^2 - 2$ on the disk $x^2 + y^2 \leq 4$.
maximum is 14, minimum is $-4$
maximum is 16, minimum is 0
maximum is 20, minimum is $-4$
maximum is 0, minimum is $-4$
maximum is 16, minimum is $-6$
Compute $\iiint_E \left(\sqrt{x^2+y^2} + z\right) dV$ where $E$ is the lower half of the solid sphere $x^2 + y^2 + z^2 \leq 9$, $z \leq 0$.
$\frac{81\pi}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$
$\frac{81\pi}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)$
$\frac{81\pi}{2}\left(\frac{\pi}{2} - \frac{1}{2}\right)$
$\frac{81\pi}{4}\left(\frac{\pi}{4} - \frac{1}{2}\right)$
$\frac{108\pi}{3}$
Determine which of the following integrals gives the area of the region in the $xy$-plane below the $x$-axis, above $y = x^2 - 2$, and to the left of $y = -2x - 2$.
$\int_{-2}^{0} \int_{-\sqrt{y+2}}^{-y/2-1} dx\, dy$
$\int_{-2}^{0} \int_{\sqrt{y+2}}^{-y/2-1} dx\, dy$
$\int_{-\sqrt{2}}^{0} \int_{x^2-2}^{0} dy\, dx$
$\int_{-\sqrt{2}}^{0} \int_{x^2-2}^{-2x-2} dy\, dx$
$\int_{-2}^{0} \int_{-\sqrt{y+2}}^{1-y/2} dx\, dy$
Determine which of the following integrals gives the volume of the region bounded by the cylinder $x^2 + y^2 = 1$, and the planes $z = 0$ and $y + z = 1$.

$\int_0^{2\pi} \int_0^1 \int_0^{1-r\sin(\theta)} r\, dz\, dr\, d\theta$
$\int_0^{2\pi} \int_0^1 \int_0^{1-r\sin(\theta)} dz\, dr\, d\theta$
$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1-y} dz\, dy\, dx$
$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{1-y} dz\, dy\, dx$
$\int_0^{\pi} \int_0^1 \int_0^{1-r\sin(\theta)} r\, dz\, dr\, d\theta$
Express the area between the ellipses $x^2 + \frac{y^2}{4} = 1$ and $x^2 + \frac{y^2}{4} = 4$ as an integral, using the substitution $x = r\cos(\theta)$ and $y = 2r\sin(\theta)$.

$\int_0^{2\pi} \int_1^2 2r dr\, d\theta$
$\int_0^{2\pi} \int_1^2 r dr\, d\theta$
$\int_0^{2\pi} \int_1^4 2r dr\, d\theta$
$\int_0^{2\pi} \int_1^4 r dr\, d\theta$
$\int_0^{2\pi} \int_1^2 dr\, d\theta$
Compute the line integral $\int_{\mathcal{C}} x\, ds$ where $\mathcal{C}$ is the curve parameterized by $\mathbf{r}(t) = t^2\mathbf{i} + t^4\mathbf{j}$, $0 \leq t \leq 1$.
$\left(5^{3/2} - 1\right)/12$
$\left(\sqrt{3} - 1\right)/6$
$\left(\sqrt{6} - 1\right)/2$
$\left(3^{5/2} - 1\right)/4$
$\left(2^{3/2} - 1\right)/3$

Let $\mathcal{C}$ be the curve $\mathbf{r}(t) = \big\langle t, \cos(2t), 1 + \sin(3t)\big\rangle$, $0 \le t \le \pi/2$, and let

$$\mathbf{F}(x, y, z) = \big\langle y(2x + z), x(x + z) - z, y(x - 1) + 2z\big\rangle$$

Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

$-\pi^2/4$

$\pi^2/2 - 1$

$1 - \pi/4$

$\pi/2$

$0$

Let $\mathcal{C}$ be the triangle with vertices $(0, 0)$, $(1, 1)$, and $(2, 0)$ oriented counterclockwise. Compute

$$\int_{\mathcal{C}} \big(\cos(x^4) + x^2y^3\big)\,dx + \big(\sin(y^4) + x^3y^2\big)\,dy$$

$0$

$2/5$

$17/30$

$-1$

$1$

Determine two vectors that are tangent to the surface $\mathbf{r}(u, v) = \langle vu^2 - 2u, uv^2 - v, uv\rangle$ at the point $(0, 1, 2)$.

$\langle 2, 1, 1\rangle$, $\langle 4, 3, 2\rangle$

$\langle 0, 1, 1\rangle$, $\langle 4, 1, 1\rangle$

$\langle 2, 4, 2\rangle$, $\langle 1, 3, 1\rangle$

$\langle 0, 2, -1\rangle$, $\langle 1, -6, 3\rangle$

$\langle 1, 2, -1\rangle$, $\langle 1, -2, 1\rangle$

Determine which of the following integrals gives the area of the surface parameterized by $\mathbf{r}(u, v) = \big\langle u\cos(v), u^2/2, u\sin(v)\big\rangle$, $0 \le u \le 1$, $0 \le v \le 2\pi$.

$\int_0^{2\pi}\int_0^1 u\sqrt{1 + u^2}\,du\,dv$

$\int_0^{2\pi}\int_0^1 \sqrt{u^2(\cos(v) + \sin(v)) + u(\sin^2(v) - \cos^2(v))}\,du\,dv$

$\int_0^{2\pi}\int_0^1 \sqrt{1 + u^2}\,du\,dv$

$\int_0^{2\pi}\int_0^1 \frac{u}{2}\sqrt{4 + u^2}\,du\,dv$

$\int_0^{2\pi}\int_0^1 \sqrt{u(\cos(v) + \sin(v)) + u^2/2}\,du\,dv$

Let $S$ be the part of the cylinder $x^2 + z^2 = 1$, with $z \ge 0$, and $0 \le y \le 1$, and let $S$ have the upward orientation. Determine which of the following equals $\iint_S \mathbf{F} \cdot \mathbf{n}\,dS$ where $\mathbf{F}(x, y, z) = e^z\mathbf{k}$.

$\int_0^1\int_0^{\pi} e^{\sin(\theta)}\sin(\theta)\,d\theta\,dy$

$\int_0^1\int_0^{\pi} -e^{\sin(\theta)}\cos(\theta)\,d\theta\,dy$

$\int_0^1\int_0^{2\pi} e^{\sin(\theta)}\cos(\theta)\,d\theta\,dy$

$\int_0^1\int_{-\pi/2}^{\pi/2} e^{\sin(\theta)}\big(\sin(\theta) - \cos(\theta)\big)\,d\theta\,dy$

$\int_0^1\int_0^{\pi/2} e^{\sin(\theta)}\sin(\theta)\,d\theta\,dy$

Let $\mathcal{C}$ be the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 2y + 3$ oriented counterclockwise when viewed from above. Calculate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \big\langle 2e^y - z, \cos(yz), xe^y\big\rangle$.

$2\pi$

$0$

$\pi\sqrt{5}$

$\pi$

$\frac{\pi}{\sqrt{5}}$

Find $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F}(x, y, z) = \left\langle xy, \frac{3}{4}y, -zy \right\rangle$ and $S$ is the surface of the sphere $x^2 + y^2 + z^2 = 4$ with the outward orientation.

$8\pi$

$\frac{32}{3}\pi$

$16\pi$

$2\pi$

$\pi$