Math 226: Calculus IV Name:____________________

Exam I *February 13, 1990* Instructor:____________________

Section:____________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 17 questions worth 6 points each.

1.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
2.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
3.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
4.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
5.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
6.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
7.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
8.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
9.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
10.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
11.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
12.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
13.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
14.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
15.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
16.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
17.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*

Score __________

1. Let

$$A = \begin{pmatrix} 2 & -3 & 4 \\ 7 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 2 & 2 \\ 6 & -1 & 1 \end{pmatrix}$$

Find $A - 3B$.

(a) $\begin{pmatrix} 5 & -9 & -2 \\ -11 & 3 & -2 \end{pmatrix}$ *(b)* $\begin{pmatrix} -1 & 3 & 10 \\ 25 & -3 & 4 \end{pmatrix}$ *(c)* $\begin{pmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & -6 & -6 \\ -18 & 3 & -3 \end{pmatrix}$ *(e) Not defined*

2. Calculate the matrix product

$$\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & -3 \\ 5 & 7 & 0 \end{pmatrix}$$

(a) $\begin{pmatrix} 2 & -2 & -3 \\ 15 & 7 & 0 \end{pmatrix}$ *(b)* $\begin{pmatrix} -8 & -13 \\ 11 & 13 \end{pmatrix}$ *(c)* $\begin{pmatrix} 17 & 1 \\ 22 & 5 \\ -3 & 6 \end{pmatrix}$

(d) $\begin{pmatrix} -8 & -13 & -3 \\ 11 & 10 & -9 \end{pmatrix}$ *(e) Not defined*

3. Which of the following matrices are in row-echelon form?

$$A = \begin{pmatrix} 1 & 3 & -2 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 & 5 \\ 1 & -3 & 2 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 2 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad E = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 4 & 5 \end{pmatrix}$$

(a) A only *(b)* A and D only *(c)* A and E only *(d)* A, C, D, and E only
(e) A, B, C, D, and E

4. Translate the following system of equations into an augmented matrix.

$$\begin{aligned} x_1 - x_5 &= -1 \\ 2x_2 - 3x_4 &= 2 \\ x_2 + x_3 + x_5 &= -5 \\ 3x_1 - 2x_3 + x_4 &= 1 \end{aligned}$$

(a) $\begin{pmatrix} 1 & 2 & 0 & -3 & -1 & : & 1 \\ 1 & 1 & 0 & 1 & 0 & : & -5 \\ 3 & 0 & -2 & 1 & 0 & : & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & -1 & : & -1 \\ 2 & -3 & : & 2 \\ 1 & 1 & : & -5 \\ 3 & -2 & : & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & -1 & : & -1 \\ 2 & -3 & 0 & : & 2 \\ 1 & 1 & 1 & : & -5 \\ 3 & -2 & 1 & : & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 & 0 & -1 & : & -1 \\ 0 & 2 & 0 & -3 & 0 & : & 2 \\ 0 & 1 & 1 & 0 & 1 & : & -5 \\ 3 & 0 & -2 & 1 & 0 & : & 1 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 & 0 & 3 & : & -1 \\ 0 & 2 & 1 & 0 & : & 2 \\ 0 & -3 & 0 & 1 & : & -5 \\ -1 & 0 & 1 & 0 & : & 1 \end{pmatrix}$

5. Which of the following is the general solution of the system of equations associated to the augmented matrix

$$\begin{pmatrix} 1 & -1 & 1 & 0 & -1 & : & 1 \\ 0 & 1 & -2 & 1 & -1 & : & 3 \\ 0 & 0 & 1 & -3 & 4 & : & 2 \end{pmatrix}$$

after back-substitution?

(a) $x_1 = 2 + 3x_4 - 4x_5$
$x_2 = 3 + 2x_3 - x_4 + x_5$
$x_3 = 1 + x_2 - x_3 + x_5$

(b) $x_1 = 1 - x_2 + x_3 - x_5$
$x_2 = 3 - 2x_3 + x_4 - x_5$
$x_3 = 2 - 3x_4 + 4x_5$

(c) $x_1 = 6 + 2x_4 - 2x_5$
$x_2 = 7 + 5x_4 - 7x_5$
$x_3 = 2 + 3x_4 - 4x_5$

(d) $x_1 = 6$
$x_2 = 7$
$x_3 = 2$

(e) $x_1 = 1 + 2x_4 - 2x_5$
$x_2 = 3 + 5x_4 - 7x_5$
$x_3 = 2 + 3x_4 - 4x_5$

6. Transform the following matrix into row-echelon form.

$$\begin{pmatrix} 1 & 1 & 4 & 0 & -1 & 10 \\ -1 & 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 3 \\ 1 & 2 & 5 & 0 & 0 & 13 \end{pmatrix}$$

(a) $\begin{pmatrix} 1 & 1 & 4 & 0 & -1 & 10 \\ 0 & 1 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & -1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 & 4 & 0 & -1 & 10 \\ 0 & 3 & 4 & -1 & -1 & 10 \\ 0 & 1 & 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 & 1 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 4 & 0 & -1 & 10 \\ 0 & 1 & -4 & 1 & 1 & -10 \\ 0 & 1 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 & 4 & 0 & -1 & 10 \\ 0 & 1 & 4 & -1 & -1 & 10 \\ 0 & 0 & 1 & 0 & 1 & 3 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 1 & 4 & 0 & 0 & 10 \\ 0 & 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$

7. Find the inverse of the following matrix, if it exists.

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & -\dfrac{1}{2} & 1 \\ \frac{1}{2} & -\dfrac{1}{2} & \frac{1}{2} \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -\dfrac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$

(d) $\begin{pmatrix} \frac{1}{2} & -\dfrac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\dfrac{1}{2} \\ -1 & 0 & 1 \end{pmatrix}$

(e) Does not exist

8. Calculate the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

(a) 0 *(b)* 2 *(c)* 4 *(d)* 8 *(e)* 16

9. Use expansion by cofactors to find the determinant of the matrix

$$\begin{pmatrix} -1 & 5 & 3 & -1 \\ 2 & 1 & 4 & 1 \\ 6 & 0 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

(a) 0 *(b)* -8 *(c)* -24 *(d)* 60 *(e)* -72

10. Calculate the (1,2)-minor, $M_{(1,2)}$, of the matrix

$$\begin{pmatrix} 3 & 1 & 2 \\ 5 & 2 & 7 \\ 1 & 4 & 2 \end{pmatrix}$$

(a) -2 *(b)* 3 *(c)* 4 *(d)* 5 *(e)* -6

11. Express $B = (1, 2, 3)$ as a linear combination of the vectors $V_1 = (1, 3, 4)$, $V_2 = (1, 0, -1)$, and $V_3 = (0, -2, -4)$, if possible, by finding constants d_1, d_2, d_3 such that $B = d_1V_1 + d_2V_2 + d_3V_3$.

(a) $d_1 = 0$, $d_2 = 1$, $d_3 = -1$ *(b)* $d_1 = 2$, $d_2 = -2$, $d_3 = 0$
(c) $d_1 = 1$, $d_2 = -5$, $d_3 = 4$ *(d)* $d_1 = -1$, $d_2 = 0$, $d_3 = 1$
(e) No solution exists

12. Find the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 7 & 4 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

(a) 0 *(b)* 1 *(c)* 2 *(d)* 3 *(e)* 4

13. Which of the following statements are always true:

(A) If V_1, V_2, V_3, V_4, V_5 are 4-dimensional vectors, then they are linearly dependent.
(B) If V_1, V_2, V_3, V_4 are linearly dependent, then V_1, V_2, V_3 are linearly dependent.
(C) If V_1, V_2, V_3, V_4 are linearly independent, then V_1, V_2, V_3 are linearly independent.
(D) If V_1, V_2, V_3 are linearly dependent, then V_1, V_2, V_3, V_4 are linearly dependent.

(a) A, B, and C only *(b)* A, C, and D only *(c)* B, C, and D only
(d) C, and D only *(e)* A, B, C, and D

14. Find all solutions to the equation $AX = 0$ where

$$A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}, \qquad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

(a) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ *(b)* $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ *(c)* $t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, t arbitrary
(d) $t\begin{pmatrix} 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, t, s arbitrary *(e) no solutions exist*

15. Determine a maximal subset of linearly independent vectors from the vectors

$$V_1 = (0,0,0,0) \quad V_2 = (1,0,0,0) \quad V_3 = (3,2,0,0) \quad V_4 = (5,4,3,0)$$

(a) V_2 *(b)* V_2, V_3 *(c)* V_2, V_4
(d) V_1, V_2, V_3 *(e)* V_2, V_3, V_4

16. Use the Gram-Schmidt process to produce an orthonormal set of vectors from the following vectors: $X_1 = (1,0,0,1)$, $X_2 = (0,2,0,1)$, $X_3 = (0,1,1,0)$.

(a) $Q_1 = (1,0,0,1)$
$Q_2 = \frac{\sqrt{6}}{6}(-1,2,0,1)$
$Q_3 = \frac{\sqrt{89}}{89}(2,0,9,-2)$

(b) $Q_1 = \frac{\sqrt{2}}{2}(1,0,0,1)$
$Q_2 = \frac{\sqrt{2}}{6}(-1,4,0,1)$
$Q_3 = \frac{\sqrt{10}}{30}(2,1,9,-2)$

(c) $Q_1 = \frac{\sqrt{2}}{2}(1,0,0,1)$
$Q_2 = \frac{\sqrt{5}}{5}(0,2,0,1)$
$Q_3 = \frac{\sqrt{2}}{2}(0,1,1,0)$

(d) $Q_1 = \frac{\sqrt{2}}{2}(1,0,0,1)$
$Q_2 = \frac{\sqrt{6}}{6}(-1,2,0,1)$
$Q_3 = \frac{1}{2}(1,1,1,-1)$

(e) $Q_1 = \frac{\sqrt{2}}{2}(1,0,0,1)$
$Q_2 = \frac{\sqrt{6}}{6}(-1,4,0,1)$
$Q_3 = \frac{\sqrt{10}}{10}(-2,-1,1,2)$

17. Suppose $y = y(x)$ is a function which satisfies the differential equation $y' + e^x y = e^x$ and $y(0) = 0$. Solve the equation to find $y(1)$.

(a) 1 *(b)* $1 - e^e$ *(c)* $e^{\frac{1}{e}}$ *(d)* $1 - e^{-e}$ *(e)* $1 - e^{1-e}$