Math 226: Calculus IV Name:________________________

Exam II *March 22, 1990* Instructor:________________________

Section:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 14 questions worth 6 points each and 6 True/False question worth 3 points each.

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1. *(a) (b) (c) (d) (e)* 8. *(a) (b) (c) (d) (e)*

2. *(a) (b) (c) (d) (e)* 9. *(a) (b) (c) (d) (e)*

3. *(a) (b) (c) (d) (e)* 10. *(a) (b) (c) (d) (e)*

4. *(a) (b) (c) (d) (e)* 11. *(a) (b) (c) (d) (e)*

5. *(a) (b) (c) (d) (e)* 12. *(a) (b) (c) (d) (e)*

6. *(a) (b) (c) (d) (e)* 13. *(a) (b) (c) (d) (e)*

7. *(a) (b) (c) (d) (e)* 14. *(a) (b) (c) (d) (e)*

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15. *(T) (F)* 18. *(T) (F)*

16. *(T) (F)* 19. *(T) (F)*

17. *(T) (F)* 20. *(T) (F)*

Score __________

1. Solve the differential equation

$$y' + y = 6e^x$$

subject to the initial condition $y(1) = 1$.

(a) $y = 3e^x$ *(b)* $y = 3e^x - 3e^{-x} + 1$ *(c)* $y = 3e^{1-x} - 3e^{x-1} + 1$ *(d)* $y = e^{x-1} - 3e^x + 3e$
(e) $y = 3e^x - 3e^{2-x} + e^{1-x}$

2. On what interval is the solution to the differential equation

$$y' - \tan(x)y = e^x, \qquad y(\pi) = 0$$

guaranteed to exist?

(a) $(-\frac{\pi}{2}, \frac{\pi}{2})$ *(b)* $(0, 2\pi)$ *(c)* $(-2\pi, 2\pi)$ *(d)* $(\frac{\pi}{2}, \frac{3\pi}{2})$ *(e)* $(-\infty, \infty)$

3. Consider the differential equation

$$(x^2 - y)^2 y' = e^x$$

Determine the region in the xy-plane where the existence of a unique solution through any specified point is guaranteed to exist.

(a) $\{(x, y) \mid y \neq x^2\}$ *(b)* $\{(x, y) \mid y \neq \pm x\}$ *(c)* all x and y
(d) $\{(x, y) \mid y \neq 0, x \neq 0\}$ *(e)* $\{(x, y) \mid (y^2 - x)^2 \neq e^x\}$

4. Solve the differential equation

$$y' = \frac{xy^2 + y}{(x^2y + x)}$$

(a) $y = cx$ *(b)* $y = c\frac{1}{x}$ *(c)* $\frac{1}{3}x^3y + xy^2 + \frac{1}{2}y^2 = c$
(d) $\frac{1}{2}x^2y^2 + xy^2 + \frac{1}{2}y^2 = c$ *(e)* $y = \frac{1}{2}x^2y^2 + c\frac{1}{x}$

5. A 1000 gal tank contains 500 gal of water having a 2% concentration of impurities. Pure water is added to the tank at a rate of 5 gal/min. At the same time the water in the tank is being filtered. The filtering process takes 10 gal/min from the tank, removes 50% of the impurities, and returns the filtered water to the tank at the same rate. The tank is kept well mixed. What is the concentration of impurities when the tank is filled to capacity?

   (a) 0.1% *(b)* 0.2% *(c)* 0.5% *(d)* 1.0% *(e)* 5.0%

6. Suppose a certain sum is deposited in a bank that pays interest at an annual rate of r compounded continuously. Find the time T required for the original sum to double in value as a function of the interest rate r.

   (a) $T = \log_2(r)$ *(b)* $T = \dfrac{\ln(r)}{\ln(2)}$ *(c)* $T = \dfrac{\ln(2)}{\ln(r)}$ *(d)* $T = \dfrac{\ln(2)}{r}$ *(e)* $T = \dfrac{r}{\ln(2)}$

7. What is the limiting value of a population $N(t)$ if $N(0) = 10$ and N satisfies the logistic equation

$$\frac{dN}{dt} = N(N^2 - 400)$$

   (a) $-\infty$ *(b)* -20 *(c)* 0 *(d)* 20 *(e)* ∞

8. Find the *stable* equilibrium solutions to the differential equation

$$\frac{dx}{dt} = 5(0.5 - x)(0.3 - x)(0.2 - x)$$

   (a) $x = 0.3$ *(b)* $x = 0.2$ and $x = 0.5$ *(c)* $x = 0.2$, $x = 0.3$, and $x = 0.5$
   (d) $x = 0$ *(e) There are no stable equilibria*

9. A body of constant mass is launched vertically upward from sea level with an initial velocity of 5000 mi/hr. Neglecting air resistance, but considering the change of gravitational attraction with altitude, x, find the maximum altitude attained by the body. The appropriate differential equation is

$$v\frac{dv}{dx} = -\frac{gR^2}{(R+x)^2}$$

Assume $R = 4000$ mi and $g = 32 \text{ ft/sec}^2 = 78545 \text{ mi/hr}^2$.

(a) 149.9 mi *(b)* 153.4 mi *(c)* 159.1 mi *(d)* 165.7 mi *(e)* 167.3 mi

10. Find the integral curve to the differential equation

$$(e^{xy}y + 3x^2y - y^2)dx + (e^{xy}x + x^3 - 2xy - 1)dy = 0$$

which passes through the point $(0, 2)$.

(a) $y = 1$ *(b)* $e^{xy} + xy(x^2 - y) - y = 0$ *(c)* $f(x, y) = e^{xy} + xy(x^2 - y) - y$
(d) $e^{xy} + xy(x^2 - y) + 1 = 0$ *(e)* $e^{xy} + xy(x^2 - y) - y + 1 = 0$

11. Determine an integrating factor μ for the differential equation

$$\tan(y)\,dx + (x + y\tan(y) - 1)\,dy = 0$$

(a) $\mu = \tan(y)\ln(x + y\tan(y) - 1)$ *(b)* $\mu = \dfrac{1}{x}$
(c) $\mu = x\sec^2(y) - x$ *(d)* $\mu = \sin(y)$ *(e)* $\mu = \cos(y)$

12. Let $\phi(x)$ be the solution to the differential equation

$$y' = \frac{y^2 + xy - x^2}{x^2}$$

satisfying $\phi(1) = 0$. Find $\phi(2)$.

(a) $\frac{3}{4}$ *(b)* $\frac{2}{5}$ *(c)* $-\dfrac{1}{3}$ *(d)* $-\dfrac{2}{3}$ *(e)* $-\dfrac{6}{5}$

13. Classify the following differential equation

$$xy' - 2y = \sqrt{x^2 - y^2}$$

(a) Linear *(b)* Separable *(c)* Exact *(d)* Homogeneous *(e)* None of the above

14. Reduce the following differential equation to a first order equation.

$$y'' + y(y')^3 = 0$$

(a) $\dfrac{dv}{dy} = -v^3$ *(b)* $\dfrac{dv}{dy} = -yv^2$ *(c)* $y' + \dfrac{1}{2}y^2(y')^3 = 0$ *(d)* $v' + yv^3 = 0$
(e) $v' + yv^3 = -v$

True/False

15. If $p(x)$ and $g(x)$ are continuous in the interval (α, β) then a solution to the equation $y' + p(x)y = g(x)$ exists and is valid on the interval (α, β).

16. If y_1 and y_2 are solutions to the differential equation $y''+p(x)y'+q(x)y = 0$, then $c_1y_1+c_2y_2$ is also a solution of the equation.

17. If y_1 and y_2 are solutions to the differential equation $y'' + p(x)y' + q(x)y = 0$, then any solution can be written as $c_1y_1 + c_2y_2$ for some choice of constants c_1 and c_2.

18. If y_1 and y_2 are solutions to the differential equation $y'' = f(x, y, y')$, then $c_1y_1 + c_2y_2$ is also a solution of the equation for some choice of constants c_1 and c_2.

19. The functions $y_1 = \sqrt{x}$ and $y_2 = \dfrac{1}{\sqrt{x}}$ are a fundamental set of solutions for the equation $xy'' + y' - \dfrac{1}{4x}y = 0$, $x > 0$.

20. There is a second order linear homogeneous differential equation such that the functions $y_1 = x^2$ and $y_2 = x^3$ form a fundamental set of solutions for all x. (Hint: check the Wronskian.)