Math 226: Calculus IV Name: ______________________

Exam III *April 24, 1990* Instructor: ______________________

Section: ______________________

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this answer sheet. There are 14 questions worth 7 points.

1.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	8.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
2.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	9.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
3.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	10.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
4.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	11.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
5.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	12.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
6.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	13.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*
7.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	14.	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*

Score ____________

1. Given that $y_1 = x^2$ is a solution of

$$2x^2y'' - 3xy' + 2y = 0$$

a second independent solution can be found of the form $y_2 = vy_1$ where v is a non-constant function. Determine the differential equation v must satisfy.

(a) $v''x^2 + 4xv' + 2v = 0$ *(b)* $v'x^2 + 2xv = 0$
(c) $v''x^2 + v'(4x + x^2) = 0$ *(d)* $2xv'' + 5v' = 0$
(e) None of the above

2. Consider the differential equation

$$4xy'' - 8y' + \frac{9}{x}y = 0, \qquad x > 0$$

with solution $y_1 = x^{3/2}$. A second solution is given by $y_2 = vy_1$ where v satisfies

$$xv'' + v' = 0$$

Find y_2.

(a) $y_2 = -x^{1/2}$ *(b)* $y_2 = x^{3/2} - \ln(x)$ *(c)* $y_2 = -x^{-1}$
(d) $y_2 = -x^{-1/2}$ *(e)* $y_2 = x^{3/2}\ln(x)$

3. Find the general solution of the homogeneous equation

$$y'' - 3y' - 4y = 0$$

(a) $y = c_1e^{4x} + c_2e^{-x}$ *(b)* $y = c_1e^{-3x} + c_2e^{-4x}$ *(c)* $y = c_1e^{-4x} + c_2e^{x}$
(d) $y = c_1e^{4x}\cos(x) + c_2e^{4x}\sin(x)$ *(e)* $y = c_1\cos 4x + c_2\sin x$

4. Find the general solution of the homogeneous equation

$$y'' + 2y' + 2y = 0$$

(a) $y = c_1 e^{-x} \cos(x) + c_2 e^{-x} \sin(x)$ *(b)* $y = c_1 e^{x} \cos(x) + c_2 e^{x} \sin(x)$ *(c)* $y = c_1 e^{2x} + c_2 e^{-x}$
(d) $y = c_1 e^{2x} + c_2 x e^{2x}$ *(e)* $y = c_1 e^{-x} + c_2 x e^{-x}$

5. Solve the initial value problem

$$y'' - 2y' + y = 0, \qquad y(0) = 1, \quad y'(0) = 3$$

(a) $y = e^{3x}$ *(b)* $y = 2e^{x} - e^{-x}$ *(c)* $y = e^{x} + 2xe^{x}$
(d) $y = e^{x} + 2x$ *(e) None of the above*

6. Use the method of undetermined coefficients to find a particular solution and then solve the initial value problem

$$y'' - y = e^{2x}, \qquad y(0) = y'(0) = 0$$

(a) $y = \frac{1}{2}e^{x} - \frac{3}{2}xe^{-x} + \frac{1}{2}e^{2x}$ *(b)* $y = -\frac{3}{4}e^{x} + \frac{1}{4}e^{-x} + \frac{1}{2}e^{2x}$ *(c)* $y = 0$
(d) $y = -\frac{1}{3}e^{x} - \frac{x}{3}e^{x} - \frac{1}{3}e^{2x}$ *(e)* $y = \frac{1}{6}e^{-x} - \frac{1}{2}e^{x} + \frac{1}{3}e^{2x}$

7. Using the method of undetermined coefficients, find the correct form for a particular solution y_p in the differential equation

$$y'' + 10y' + 25y = x^2 + e^x$$

(a) $y_p = x(Ax^2 + Bx + C) + De^x$ *(b)* $y_p = ax^2 + Bx + C + De^{-5x} + Exe^{-5x}$
(c) $y_p = Ax^2 + Be^x$ *(d)* $y_p = Ax^2 + Bx + C + De^x$
(e) $y_p = Ax^2 + Bx + C + Dxe^x$

8. Let $y(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ be the solution of

$$y'' + y = \sin(x)\cos(x)$$

given by the method of variation of parameters. Find u_1.
(a) $u_1 = \tan(x)$ *(b)* $u_1 = -\cot(x)$ *(c)* $u_1 = -\sin^2(x)\tan(x)$
(d) $u_1 = -\frac{1}{3}\sin^3(x)$ *(e)* $u_1 = \frac{1}{2}\cos^2(x)$

9. The displacement in a certain spring-mass system with no external force and no damping satisfies the initial value problem

$$m\ddot{u} + ku = 0, \qquad u(0) = u_0(\neq 0), \quad \dot{u}(0) = 0$$

For what values of time T is the displacement zero?
(a) $T = (2j+1)\sqrt{\frac{m}{k}}\frac{\pi}{2}, \quad j = 0, 1, 2, \ldots$ *(b)* $T = (2j+1)\frac{\pi}{2u_0}, \quad j = 0, 1, 2, \ldots$
(c) $T = (2j+1)\sqrt{\frac{m}{ku_0}}\pi, \quad j = 0, 1, 2, \ldots$ *(d)* $T = 2j\sqrt{\frac{m}{k}}\frac{1}{u_0}, \quad j = 0, 1, 2, \ldots$
(e) $T = j\sqrt{\frac{m}{k}}\pi, \quad j = 0, 1, 2, \ldots$

10. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n^2}{2^n} x^n$$

(a) 0 *(b)* 1/2 *(c)* 1 *(d)* 2 *(e)* ∞

11. Compute the Taylor series for $f(x) = \dfrac{1}{2-x}$ about $x_0 = 1$.

(a) $1 + (x-1) + (x-1)^2 + \dots$

(b) $\dfrac{1}{2} + \dfrac{1}{2^2}(x-1) + \dfrac{1}{2^3}(x-1)^2 + \dots$

(c) $1 - x + x^2 - x^3 + \dots$

(d) $1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots$

(e) $\dfrac{1}{2} + \dfrac{1}{2^2}x + \dfrac{1}{2^3}x^2 + \dots$

12. Assuming $a_0 = 1$, determine a_n for $n > 1$ so that

$$\sum_{n=1}^{\infty} n a_n x^{n-1} - 3 \sum_{n=0}^{\infty} a_n x^n = 0$$

(a) $a_n = \dfrac{1}{n!}$ *(b)* $a_n = \dfrac{3}{n}$ *(c)* $a_n = \dfrac{3}{n!}$ *(d)* $a_n = \dfrac{3^n}{n}$ *(e)* $a_n = \dfrac{3^n}{n!}$

13. Find the recurrence relation for the coefficients of a power series solution about $x_0 = 0$ of the differential equation $y'' + xy' + y = 0$.

(a) $a_{n+2} = \frac{1}{n} a_n$ *(b)* $a_{n+2} = -\frac{1}{n+2} a_n$ *(c)* $a_n = -\frac{1}{n+2} a_{n+2}$

(d) $a_{n+2} = -\frac{1}{(n+2)(n+1)} a_n$ *(e)* $a_n = \frac{1}{(n+2)(n+1)} a_{n+2}$

14. Determine the lower bound for the radius of convergence of a power series solution about $x_0 = -1$ of the differential equation $(x^2 + 2x - 3)y'' - y = 0$.

(a) $1/\sqrt{3}$ *(b)* 1 *(c)* 2 *(d)* $\sqrt{3}$ *(e)* ∞