Math 226: Calculus IV

Final Exam *May 11, 1990*

Name:________________________

Instructor:________________________

Section:________________________

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this answer sheet. There are 25 questions worth 6 points each.

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25. *(a) (b) (c) (d) (e)*

Score ____________

1. Let

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 5 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -1 \\ 1 & 2 \\ 1 & 0 \end{pmatrix}$$

Compute the matrix product AB.

(a) $\begin{pmatrix} 3 & -2 \\ 0 & 10 \end{pmatrix}$ *(b)* $\begin{pmatrix} 5 & 3 \\ 6 & 10 \end{pmatrix}$ *(c)* $\begin{pmatrix} 5 & 4 & 0 \\ 6 & 10 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & -1 \\ 6 & 10 \\ 0 & 0 \end{pmatrix}$ *(e) Not defined*

2. Find the general solution to the following system of linear equations.

$$\begin{array}{rcrcrcr} 2x_1 & - & 2x_2 & + & 6x_3 & = & -4 \\ x_1 & + & x_2 & + & 2x_3 & = & 0 \\ -3x_1 & + & 3x_2 & - & 9x_3 & = & 1 \end{array}$$

(a) $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ *(b)* $x_1 = 0$, $x_2 = 0$, $x_3 = t$ *(c)* $x_1 = 2 + 2t$, $x_2 = 4 + 5t$, $x_3 = t$ *(d)* $x_1 = -2 - 2t$, $x_2 = t$, $x_3 = t$ *(e) System is inconsistent*

3. Calculate the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 2 & -1 \\ 2 & -3 & 1 & 5 \\ 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) 14 *(b)* -120 *(c)* 480 *(d)* -24 *(e)* 0

4. The vectors resulting from the Gram-Schmidt process applied to $X_1 = (1, 1, 0, 1)$ and $X_2 = (0, 1, 1, 1)$ are

(a) $Q_1 = (1, 1, 0, 1), \quad Q_2 = (-1, 1, 1, 0)$

(b) $Q_1 = \frac{1}{\sqrt{3}}(1, 1, 0, 1), \quad Q_2 = \frac{1}{\sqrt{3}}(-1, 1, 1, 0)$

(c) $Q_1 = (1, 0, 0, 0), \quad Q_2 = (0, 1, 0, 0)$

(d) $Q_1 = \frac{1}{\sqrt{3}}(1, 1, 0, 1), \quad Q_2 = \frac{3}{\sqrt{7}}(-\frac{2}{3}, \frac{1}{3}, -1, \frac{1}{3})$

(e) $Q_1 = \frac{1}{\sqrt{3}}(1, 1, 0, 1), \quad Q_2 = \frac{1}{\sqrt{15}}(-2, 1, 3, 1)$

5. Find an integrating factor for the differential equation

$$y' - \frac{2}{x}y = x^3$$

(a) x *(b)* x^2 *(c)* x^{-2} *(d)* x^{-1} *(e)* $2\ln(x)$

6. Which function below is an integrating factor of

$$\frac{1}{x}e^{xy}\left(1 + y - \frac{1}{x}\right)dx + e^{xy}dy = 0$$

(a) $\mu = x^2$ *(b)* $\mu = e^x$ *(c)* $\mu = xe^y + ye^x$ *(d)* $\mu = e^{y^2}$ *(e)* $\mu = e^{x+y}$

7. Solve the initial value problem

$$xy' + y = \sin(x), \qquad y(\pi/2) = 2$$

(a) $y = x^{-1}\cos(x) + 2$ *(b)* $y = x^{-1}\cos(x) + \pi x^{-1}$ *(c)* $y = -x^{-1}\cos(x) + \pi x^{-1}$
(d) $y = x^{-1}\sin(x) + \dfrac{(2\pi - 2)}{\pi}$ *(e)* $y = \pi x^{-1}\sin(x)$

8. Find the solution to the initial value problem

$$y' + 2xy = x, \qquad y(0) = 0$$

(a) $y = \frac{1}{2}\left(1 - e^{-x^2}\right)$ *(b)* $y = e^{x^2} - 1$ *(c)* $y = 2x + xe^{-x^2}$
(d) $y = x^2 + x\left(e^{x^2} - 1\right)$ *(e) None of the above*

9. The critical points of the differential equation

$$\frac{dN}{dt} = N(N^2 - 5N + 6)$$

are $N_0 = 0$, 2, and 3. Their stability properties are as follows:
(a) $N_0 = 0$ unstable, $N_0 = 2$ unstable, $N_0 = 3$ stable
(b) $N_0 = 0$ unstable, $N_0 = 2$ stable, $N_0 = 3$ unstable
(c) $N_0 = 0$ stable, $N_0 = 2$ stable, $N_0 = 3$ stable
(d) $N_0 = 0$ stable, $N_0 = 2$ unstable, $N_0 = 3$ unstable
(e) $N_0 = 0$ unstable, $N_0 = 2$ unstable, $N_0 = 3$ unstable

10. Determine the largest region in the plane where the existence of a unique solution through any point is guaranteed for the differential equation

$$y' = \frac{\ln|xy|}{x^2 - y^2}$$

(a) $\{(x,y)\,|\,x \neq \pm y\}$ *(b)* $\{(x,y)\,|\,x > 0, y > 0, x \neq y\}$ *(c)* $\{(x,y)\,|\,x > y, y > 0\}$
(d) $\{(x,y)\,|\,x \neq \pm y, x \neq 0, y \neq 0\}$ *(e) None of the above*

11. The solution of

$$(1 + y\cos(xy))dx + x\cos(xy)dy = 0$$

is given implicitly by
(a) $y + \cos(xy) = c$ *(b)* $2xy + \tan(xy) = c$ *(c)* $3x + y\sin(xy) = c$
(d) $\sin(x)\cos(y) = c$ *(e)* $x + \sin(xy) = c$

12. Solve the differential equation

$$y' = \frac{y^4 + x^4}{xy^3}$$

by making a substitution $v = \dfrac{y}{x}$.

(a) $y = c\ln(x) + \dfrac{1}{x^4}$ *(b)* $y^4 = x^4 + c\ln(x)$
(c) $y^3 = cx^3$ *(d)* $\ln(y) = x^4 + c$
(e) $y^4 = x^4(\ln(x^4) + c)$

13. Suppose that a certain sum of money is deposited in a bank that pays interest at an annual rate of 8% compounded continuously. Find the time T required for this original sum to triple.

(a) 8.66 years *(b)* 10.14 years *(c)* 11.45 years *(d)* 12.64 years *(e)* 13.73 years

14. Each of the following is a set of solutions to $y'' - y = 0$. Find the one which is **not** a fundamental set of solutions.

(a) $\{e^x, e^{-x}\}$ *(b)* $\{e^x + e^{-x}, e^x - e^{-x}\}$ *(c)* $\{e^{1-x}, e^{1+x}\}$
(d) $\{e^{x-1}, e^{x+1}\}$ *(e)* $\{e^x, 2e^{-x}\}$

15. Compute the Wronskian of the functions $y_1 = e^{2x}$ and $y_2 = xe^{2x}$

(a) $e^{2x}(1 + 2x)$ *(b)* $e^{4x}(1 + 2x)$ *(c)* e^{4x} *(d)* $2xe^{2x}$ *(e)* e^{2x}

16. Solve the differential equation with initial conditions

$$y'' + 2y' + 5y = 0, \qquad y(0) = 1, \quad y'(0) = 1$$

(a) $y = e^{-x}(\cos(2x) + \sin(2x))$ *(b)* $y = e^{-x}(\cos(x) + 2\sin(x))$
(c) $y = e^{-x}(\cos(\sqrt{5}x) + \frac{2}{\sqrt{5}}\sin(\sqrt{5}x))$ *(d)* $y = e^{-2x}(\cos(2x) - \frac{1}{2}\sin(2x))$
(e) None of the above

17. Given that $y_1 = x^{-1}$ is a solution of

$$x^2y'' + 3xy' + y = 0$$

find another independent solution of the form $y_2 = vy_1$.

(a) $y_2 = x^{-1}(\cos(|\ln(x)|) + \sin(|\ln(x)|))$ *(b)* $y_2 = x$ *(c)* $y_2 = x\ln(x)$

(d) $y_2 = x^{-1}\ln(x)$ *(e)* $y_2 = x^{-1}\sum_{n=0}^{\infty}(n+1)x^n$

18. The form of the particular solution of

$$y'' - 2y' + y = x + 1 + xe^x$$

given by the method of undetermined coefficients is

(a) $y_p = Ax + xe^x$ *(b)* $y_p = Ae^x + Bxe^x$ *(c)* $y_p = Ax^2e^x$

(d) $y_p = Ax + B + x(Cx + D)e^x$ *(e)* $y_p = Ax + B + x^2(Cx + D)e^x$

19. The particular solution of

$$y'' + y = \tan(x)$$

given by the method of variation of parameters is

(a) $y_p = \ln|\tan(x)|$ *(b)* $y_p = (\tan(x) + 1)\cos(x)$ *(c)* $y_p = -\cos(x)\ln|\tan(x) + \sec(x)|$

(d) $y_p = \tan^2(x) + 1$ *(e) None of the above*

20. Compute the radius of convergence of the series expansion

$$\frac{1}{3+x^2} = \frac{1}{3} - \frac{1}{3^2}x^2 + \frac{1}{3^3}x^4 - \frac{1}{3^4}x^6 + \dots$$

(a) 1 *(b)* $\sqrt{3}$ *(c)* 3 *(d)* $\frac{1}{3}$ *(e)* ∞

21. Determine the power series expansion around $x_0 = 0$ for $f(x) = \dfrac{1}{(1+x)^2}$.

(a) $1 - 2x + 3x^2 - 4x^3 + \dots$

(b) $1 + x^2 + x^4 + x^6 + \dots$

(c) $1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x^3 + \dots$

(d) $1 - \frac{1}{2}x^2 + \frac{1}{4}x^4 - \frac{1}{6}x^6 + \dots$

(e) $1 - x^2 + x^4 - x^6 + \dots$

22. Find a power series solution around $x_0 = 0$ for the differential equation

$$y'' - xy' - 2y = 0$$

which satisfies $y(0) = 0$, $y'(0) = 1$.

(a) $y = x + \frac{1}{3 \cdot 2}x^3 + \frac{1}{5 \cdot 4}x^5 + \frac{1}{7 \cdot 6}x^7 + \dots$

(b) $y = x - \frac{1}{2}x^3 + \frac{1}{4 \cdot 2}x^5 - \frac{1}{6 \cdot 4 \cdot 2}x^7 + \dots$

(c) $y = x + \frac{1}{3}x^3 + \frac{1}{5 \cdot 3}x^5 + \frac{1}{7 \cdot 5 \cdot 3}x^7 + \dots$

(d) $y = x - \frac{1}{2}x^3 + \frac{1}{4 \cdot 2}x^5 - \frac{1}{6 \cdot 4 \cdot 2}x^7 + \dots$

(e) $y = x + \frac{1}{2}x^3 + \frac{1}{4 \cdot 2}x^5 + \frac{1}{6 \cdot 4 \cdot 2}x^7 + \dots$

23. Find a fundamental set of solutions for the following Euler equation

$$x^2y'' - xy' - 3y = 0$$

(a) $\{x^3, x^3\ln(x)\}$

(b) $\{x^3, x^{-1}\}$

(c) $\{x^3, x^{-1}\ln(x)\}$

(d) $\{x, x^{-3}\}$

(e) $\left\{\sum_{n=0}^{\infty}\frac{1}{n!}x^{n+1}, \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}x^{n-1}\right\}$

24. The following differential equation has a regular singular point at $x_0 = 0$.

$$x^2y'' + 4xy' + (2 + x^2)y = 0$$

Which of the following functions is a solution to this equation.

(a) $y = x^{-1}(1 - \frac{1}{3!}x^2 + \frac{1}{5!}x^4 - \ldots)$

(b) $y = (x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \ldots)$

(c) $y = x^{-1}(1 + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \ldots)$

(d) $y = x^{-3}(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \ldots)$

(e) $y = x^{-2}(1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \ldots)$

25. The differential equation

$$x^2y'' + \sin(x)y' - 4y = 0$$

is guaranteed to have solution(s) of the form

(a) $y = |x|^{-2}\left(1 + \sum_{n=1}^{\infty} a_n x^n\right)$

(b) $y = |x|^{-1}\left(1 + \sum_{n=1}^{\infty} a_n x^n\right)$

(c) $y = \left(1 + \sum_{n=1}^{\infty} a_n x^n\right)$

(d) $y = |x|^{2}\left(1 + \sum_{n=1}^{\infty} a_n x^n\right)$

(e) $y = |x|^r$ for some r