Solutions to Math. 226, Exam 2

1a. The characteristic equation is $4r^2 - 8r + 3 = 0$ and has roots $r = 12or32$. Therefore the general solution to the DE is given by $y = c_1e^{12x} + c_2e^{32x}$.

1b. The characteristic equation is $r^2 + 6r + 25 = 0$ and has roots $r = -3 \pm 4i$. Therefore, the general solution of the D.E. is given by $y = e^{-3x}[c_1 \cos 4x + c_2 \sin 4x]$.

2. Another solution to the DE is given by $y = e^x v$, v to be determined. We must have

$$x(e^x v)'' - (2x + 1)(e^x v)' + (x + 1)e^x v = 0$$

or

$$x(e^x v'' + 2e^x v' + e^x v) - (2x + 1)(e^x v' + e^x v) + (x + 1)e^x v = 0$$

or

$$xe^x v'' - e^x v' = 0 or v'' - 1xv' = 0.$$

If we let $u = v'$ then we obtain the equation $u' - 1xu = 0$ or $duu = dxx$ or $\ln u = \ln x$ or $u = x$ or $v' = x$ or $v = 12x^2$. Therefore $y_2 = 12x^2e^x$ is a second solution which is independent from $y_1 = e^x$. Thus the general solution to the above D.E. is given by $y = e^x[c_1 + c_2x^2]$.

3. The general solution to the DE is given by

$$y = c_1x + c_2 1x + y_p,$$

where $y_p = y_1u_1 + y_2u_2$ and u_1 and u_2 satisfy the system

$$\{ y_1 u_1' + y_2u_2' = 0$$

or

$$a_{n+2} = (n + 2)a_n(n + 2)(n + 1) \ or \ a_{n+2} = 2n + 1a_n, n = 0, 1, 2...$$

5b. Since $y(0) = 0$ we have $a_0 = 0$ and since $y'(0) = 1$ we have $a_1 = 1$. By the recurrence relation we see that $a_{2n} = 0 \ and \ a_{2n+1} = 1n!$.

5c. The solution is $y = \sum_{n=0}^{\infty} 1n!x^{2n+1}$. Since $1(n + 1)!|x|^{2(n+1)+1}1n!|x|^{2n+1} = x^2n + 1 \longrightarrow 0 as n \to \infty$, for all $x \in R$ we have that the power series converges for all $x \in R$, i.e. its radius of convergence is $R = \infty$.