amsppt

Mathematics 226
Solutions to Test 2

1a) We have:

$$r^2 + 4r + 29 = 0 \Longrightarrow r = -4 \pm \sqrt{16 - 116}2 \Longrightarrow r = -2 \pm 5i$$

Thus the general solution of the DE is: $y = e^{-2x}[c_1 cos5x + c_2 sin5x]$

1b) We have:

$$r^2 + 12r + 64 = 0 \Longrightarrow (r+6)^2 = 0 \Longrightarrow r = -6$$

Thus the general solution of the DE is: $y = c_1 e^{-6x} + c_2 x e^{-6x}$

2a) We have $r^2 - 9 = 0 \Longrightarrow r = \pm 3$. The solution to $y'' - 9y = 0$ is $y_c = c_1 e^{3x} + c_2 e^{-3x}$. A particular solution is of the form: $y_p = Ax^2 + Bx + C$. If we substitute y_p into the DE we obtain: $2A - 9(Ax^2 + Bx + C) = 18x^2 + 5$ Therefore we have

$$-9A = 18,\ -9B = 0 and 2A - 9C = 5$$

By solving these equations we obtain: $A = -2,\ B = 0$ and $C = -1$. Thus the general solution is: $y = c_1 e^{-3x} + c_2 e^{-3x} - 2x^2 - 1$

2b) We have: $1 = y(0) = c_1 + c_2 - 1 \Longrightarrow c_1 + c_2 = 2$. Since
$y'(x) = 3c_1 e^{3x} - 3c_2 e^{-3x} - 4x$ we have $0 = y'(0) = 3c_1 - 3c_2 \Longrightarrow c_1 - c_2 = 0$.
Therefore $c_1 = c_2 = 1$ and the solution is: $y = e^{3x} + e^{-3x} - 2x^2 - 1$

3) A particular solution is of the form

$$y_p = y_1 u_1 + y_2 u_2,\ y_1 = e^x,\ y_2 = xe^x, where$$

$\{y_1 u_1' + y_2 u_2' = 0$
Thus $u_1' = 0x$
1 x
=-1 and u'$_2$ = 10
Thus $y_p = -xe^x + xe^x lnx$ and the general solution is: $y = c_1 e^x + c_2 xe^x + xe^x lnx$

4) We have $mu'' + ku = 0$. Therefore $u'' + kmu = 0$. Since $k\Delta l = W$ we have $u'' + Wm.\Delta lu = 0$. Therefore $u'' + g\Delta lu = 0$. since $g = 32$ and $\Delta l = 12$ we have $u'' + 64u = 0$. Since $r^2 + 64 = 0$ implies $r = \pm i8$ we have $u = c_1 cos8t + c_2 sin8t$. Since $0 = u(0) = c_1$ we have $u' = 8c_2 sin8t$. Thus $u'(0) = 16 = 8c_2$ and $c_2 = 2$. Therefore: $u(t) = 2sin8t$. The period is $T = 2\pi 8 = \pi 4$. Thus the time for 4 cycles is: $\pi\ sec$

5a) We have $y' = \sum_{n=1}^{\infty} na_n x^{n-1}$ and $y'' = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$. Thus we must have:

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + (x-1)\sum_{n=1}^{\infty} na_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

or

$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n + \sum_{n=0}^{\infty} na_n x^n - \sum_{n=0}^{\infty}(n+1)a_{n+1}x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

or

$$\sum_{n=0}^{\infty}[(n+2)(n+1)a_{n+2} + na_n - (n+1)a_{n+1} + a_n]x^n = 0$$

or

$$(n+2)(n+1)a^{n+2} + na_n - (n+1)a_{n+1} + a_n] = 0$$

or

$$(n+2)(n+1)a_{n+2} - (n+1)a_{n+1} + (n+1)a_n = 0$$

or

$$a_{n+2} = a_{n+1} - a_n n + 2$$

5b) We have $a_2 = 12(a_1 - a_0)$ and $a_3 = 13(a_2 - a_1) = 13(12a_1 - 12a_0 - a_1)$ or

$$a_3 = -16(a_0 + a_1)$$