amsppt

Name: ______________________ **Professor:** A.Alexandrou Himonas■

Mathematics 226
Differential Equations and Linear Algebra
Fall Semester
Final Exam
December 20, 1991, 8:00-10:00 AM

This Examination contains eight problems worth a total of 150 points, each problem worth 20 points,except the last which worths 10 points, on 10 sheets of paper including the front cover and one extra sheet on the back. Do all your work in this booklet and show your computations. Calculators, books and notes are not allowed.

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Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

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GOOD LUCK and HAPPY HOLIDAYS

1. (a) Solve the initial value problem $y' - xy + 2x = 0, y(0) = 0$.

Answer: ______________________________

(b) Solve the DE $(xe^{xy} + 4y^3)dydx = 2x - ye^{xy}$

Answer: ______________________________

2. Given that $y_1 = e^x$ and $y_2 = x^2e^x$ are two solutions to the DE

$$xy'' - (2x + 1)y^1 + (x + 1)y = 0,$$

find the general solution to the DE $xy'' - (2x + 1)y' + (x + 1)y = x(x^2 - 2x - 1)e^x$.

Answer: ______________________________

3. Let $y(x) = \sum_{n=0}^{\infty} a_n (x-3)^n$ be the power series expansion of the solution to the initial value problem $y'' - 2(x-3)y' - 2y = 0, y(3) = 0, y'(3) = 1$.

a) Find the recurrence relation for the coefficients a_n.

b) Then compute all a_n and write the solution $y(x)$. What is its radius of convergence?

4. Let

$$A = (\,1\,)\,3 - 52 - 7$$

a) Find the reduced echelon form of A.

b) Find the solution space W of the system $Ax = 0$ and a basis for W.

c) Find the rank of A. Then find $dim(W)$.

5. a) By row elementary operations A are reduced to

$$(\,1\,)\,3 - 52 - 7$$

$$(\,1\,)\,3 - 501$$

b) The system $Ax = 0$ is reduced to

$$x_1 \quad - 2x_3 \quad - 5x_5 = 0$$

Let $x_5 = \alpha$, $x_3 = \beta$ then $x_4 = 3\alpha$, $x_2 = \beta - 2\alpha$ and $x_1 = 5\alpha + 2\beta$ and

$$W = \{(\,x\,)_1$$

A basis for W is $\{(\,5\,)$

5. Use Cramer's Rule to solve the system:

$$3x_1 + x_2 \quad = 1$$

6. Find the general solution of the system of DE

$$dxdt = (\,1\,)\,2$$

7. Assume that the human population $p = p(t)$ of the earth obeys the logistic equation

$$dpdt = r(1 - PK)P, \; r = 0.029, \; K = 10^{10}.$$

If the population now is 1210^{10}, find the population $P(t)$ at any time t. Also find $\lim_{t \to \infty} P(t)$.

8. Find all terms up to degree four of the Taylor Series expansion $a + x_0 = 0$ of the function $y = y(x)$, where y is the solution to the following initial value problem

$$y'' - xy' - e^{2y} = 0, \;\; y(0) = 0, \;\; y'(0) = 1.$$