Math 226: Calculus IV   Name:_____

Exam II   *March 21, 1991*   Score:_____

1in Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Find the general solution of the differential equation

$$xy' + 2y = \sin(x)$$

$y = \frac{1}{x^2}[\sin(x) - x\cos(x) + c]$ $y = \frac{1}{x}[\sin(x) + x\cos(x) + c]$ $y = \frac{1}{x^2}[\sin(x) + \cos(x) + c]$ $y = cx[\sin(x) - x\cos(x)]$
$y = cx^2[\sin(x) - x\cos(x)]$

Suppose $y(0) = 0$ and

$$y' - y = x^3 e^x$$

Find $y(1)$. $e/4$ e 1 $4e$ e^{-1}

Find the region in the xy-plane where there exists a unique solution through any specified point for the differential equation

$$y' = (x^2 + y^2 - 1)^{1/3}$$

$\{(x,y) \mid x^2 + y^2 < 1 \text{ and } x^2 + y^2 > 1\}$ $\{(x,y) \mid x^2 + y^2 < 1\}$ $\{(x,y) \mid x^2 + y^2 > 1\}$ $\{(x,y) \mid y^2 \neq 0\}$ *everywhere*

On what interval is the solution to the following initial value problem valid?

$$y' = -2\frac{x}{y} \qquad y(0) = 1$$

$-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$ $x < \frac{1}{2}$ $-\frac{1}{2} < x < \frac{1}{2}$ $x > 0$ $x < \frac{\sqrt{2}}{2}$

Determine the integral curve of the differential equation

$$\frac{dy}{dx} = y + \frac{1}{y}$$

which passes through the point $(0,2)$. $5e^{2x} - y^2 = 1$ $y^2 = 2x + 4$ $\ln|y^2 + 1| = \frac{1}{2}x^2 + \ln(5)$ $\frac{1}{2}y^2 + \ln(y) = x + 2 + \ln(2)$ $\sqrt{y^2 + 1} = x + \sqrt{5}$

A bank offers two types of certificates of deposit. The first type pays interest at 7% per year compounded continuously. The second type pays interest at 8% per year, but requires an initial fee of 2% deducted from the amount deposited. Suppose an equal amount of money is deposited in each type of certificate. At what time will the two certificates be worth the same amount? 2.02 years 0 years 1.57 years 3.11 years 0.98 years

The water in a 200 gal aquarium is to be replenished by first draining out 100 gal and then refilling the aquarium with fresh water at a rate of 2 gal/min. At the same time, the mixture continues to drain out of the tank at a rate of 1 gal/min. If the aquarium originally had 5% impurities in the water, what will the concentration of impurities be when the the tank is full? 1.25% 0.05% 2.71% 0.47% 3.52%

Classify the equilibrium solutions of the logistic equation

$$\frac{dN}{dt} = 100 - 25N + N^2$$

$N = 5$ stable $N = 0$ stable $N = 5$ unstable $N = 0$ unstable $N = 0$ unstable
$N = 20$ unstable $N = 10$ unstable $N = 20$ stable $N = 5$ stable $N = 10$ unstable
                                                            $N = 20$ unstable $N = 20$ stable

A 12 kg body falls from rest in a medium offering a resistance equal to 0.75 times the square of the velocity. Find the limiting velocity of the body. 12.52 m/sec 10.02 m/sec 9.13 m/sec 8.66 m/sec 7.41 m/sec

Solve the differential equation

$$(3x^2 - 2xy + 2)\,dx + (3y^2 - x^2 + 1)\,dy = 0$$

subject to the initial condition $y(0) = 1$. $x^3 + y^3 - x^2 y + 2x + y = 2$ $f(x,y) = x^3 + y^3 - x^2 y + 2x + y$ $x^3 - x^2 y + 2x = 0$ $f(x,y) = x^3 - x^2 y + 2x$ $y^3 - x^2 y + y = 2$

Find an integrating factor for the differential equation

$$(y + e^{y-x})\, dx + (1 + xe^{y-x})\, dy$$

e^x $x\, e^{-y}$ xe^x $1 - e^{-y}$

Solve the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

$\tan^{-1} \frac{y}{x} = \ln\sqrt{x^2 + y^2} + c$ $(x-y)x^3 = (\ln(x) + c)(x^2 + y^2)$ $\tan^{-1} \frac{y}{x} = \ln(x) + c$ $y^2 = cx^4 - x^2$ $y = cx^2 - x$

Reduce the following second order equation to a first order equation.

$$y^2 y'' + y(y')^2 = y'$$

$\frac{dv}{dy} + \frac{1}{y} v = \frac{1}{y^2}$ $y^2 v' + yv^2 = v$ $y^2 v'' + y(v')^2 = v'$ $y^2 \frac{dv}{dy} + yv^2 = 1$ $\frac{dv}{dy} + \frac{1}{y} v^2 = \frac{1}{y^2}$

Classify the differential equation

$$(x - \cos(y))y' + x\sin(y) = 0$$

None of the above Linear Separable Exact Homogeneous

Solve the initial value problem

$$y'y'' - 6x^2 = 0 \qquad y(1) = y'(1) = 2$$

and evaluate $y(2)$. 5.73 6.85 1.23 4.08 3.41