Math 226: Calculus IV Name:__________

Exam III *April 25, 1991* Score:__________

1in Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Compute the Wronskian of the functions e^x and xe^x. e^{2x} $e^{2x}+xe^{2x}$ e^x+xe^x $e^{2x}-xe^{2x}$ 0

Determine which of the following pairs of functions are **NOT** linearly independent. $\frac{x}{1-x}$, $1-\frac{1}{1-x}$ $\cos(x)$, $\sin(x)$ e^x, e^{2x} x^2, $x-1$ $\cos(x)$, $\cos(3x)$

Given that $y_1 = x^3$ is a solution of

$$xy'' - y' - \frac{3}{x}y = 0, \qquad x > 0$$

find a second linearly independent solution of the form $y_2 = vy_1$ for some non-constant function v. $y_2 = \frac{1}{x}$ $y_2 = \frac{1}{x^4}$ $y_2 = x^3 e^{-\frac{5}{4}x}$ $y_2 = e^{-\frac{5}{4}x}$ $y_2 = x^3\ln(x)$

Find the general solution of

$$4y'' + 4y' + y = 0$$

$y = e^{-\frac{1}{2}x}(c_1 + c_2 x)$ $y = e^{-\frac{1}{2}x}(c_1 + c_2\sqrt{x})$ $y = c_1 e^{-\frac{1}{2}x} + c_2\sqrt{x}$ $y = c_1 e^{-\frac{1}{2}x} + c_2 x$ $y = c_1 e^{-x} + c_2 e^{-\frac{1}{2}x}$

Solve the initial value problem

$$y'' - 2y' - 3y = 0 \qquad y(0) = 0, \quad y'(0) = 1$$

$y = \frac{1}{4}e^{3x} - \frac{1}{4}e^{-x}$ $y = -\frac{1}{4}e^{3x} + \frac{1}{4}e^{x}$ $y = \frac{1}{3}e^{3x}$ $y = -e^{-x}$ $y = \frac{1}{3}e^{3x} - e^{x}$

Find the general solution to

$$2y'' + 10y' + 17y = 0$$

$y = e^{-\frac{5}{2}x}\left(c_1\cos(\frac{3}{2}x) + c_2\sin(\frac{3}{2}x)\right)$ $y = c_1\cos(\frac{3}{2}x) + c_2\sin(\frac{3}{2}x)$ $y = c_1 e^{-\frac{5}{2}x} + c_2 e^{\frac{3}{2}x}$ $y = c_1 e^{-2x}\cos(2x) + c_2 e^{-2x}\sin(2x)$ $y = c_1 e^{-2x} + c_2 x e^{-2x}$

Determine a suitable form for a particular solution y_p of the equation

$$y'' + y' = x^2 + xe^x$$

$y_p = x(Ax^2+Bx+C) + (Dx+E)e^x$ $y_p = (Ax^2+Bx+C) + (Dx+E)e^x$ $y_p = Ax^2 + x(Bx+C)e^x$ $y_p = x^2(Ax^2+Bx+C) + Dxe^x$ $y_p = x(Ax^2+Bx+C+Dxe^x)$

Find a particular solution y_p to the differential equation

$$y'' - 3y' = \cos(x)$$

$y_p = -\frac{1}{10}\cos(x) - \frac{3}{10}\sin(x)$ $y_p = 1 + e^{3x}$ $y_p = 1 + e^{3x} + \cos(x)$ $y_p = -\frac{1}{3}\cos(x) + \frac{2}{3}\sin(x)$ $y_p = \frac{1}{15}\cos(x) + \frac{8}{15}\sin(x)$

Let $y(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ be a particular solution of

$$y'' + y = \tan(x), \qquad 0 < x < \frac{\pi}{2}$$

given by the method of variation of parameters. Find u_2. $u_2 = -\cos(x)$ $u_2 = \sin(x)$ $u_2 = \sec^2(x)$ $u_2 = -\sec(x)$ $u_2 = \cot(x)$

A 3 lb weight stretches a spring 6 inches. The weight is hit from below giving it an upward velocity of 1 ft/sec. Find the resulting displacement u of the weight from equilibrium. Assume the positive u direction is *down* and neglect air resistance. $u = -\frac{1}{8}\sin(8t)$ $u = \frac{1}{8}\cos(8t)$ $u = -\frac{1}{4\sqrt{3}}\sin(4\sqrt{3}t)$ $u = \frac{1}{4\sqrt{3}}\cos(4\sqrt{3}t)$ $u = -\cos(8t) + \sin(8t)$

Which of the following is the power series for e^x centered at $x_0 = 1$: $e + e(x-1) + \frac{e}{2!}(x-1)^2 + \frac{e}{3!}(x-1)^3 + \ldots$ $1 + (x-1) + \frac{1}{2!}(x-1)^2 + \frac{1}{3!}(x-1)^3 + \ldots$ $1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots$ $e - e(x-1) + \frac{e}{2!}(x-1)^2 - \frac{e}{3!}(x-1)^3 + \ldots$ $1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \ldots$

Determine the radius of convergence of the series

$$\sum_{n=0}^{\infty}(-1)^n \frac{1}{4^{n+1}} x^{2n}$$

2 4 1 $\frac{1}{4}$ ∞

Find the recurrence relation for the coefficients of a power series solution centered at $x_0 = 0$ of the differential equation

$$(1-x)y'' + y = 0$$

$a_{n+2} = \frac{n}{n+2} a_{n+1} - \frac{1}{(n+2)(n+1)} a_n$ $a_{n+2} = \frac{n}{n+2} a_{n+1} - \frac{1}{n+2} a_n$ $a_{n+2} = \frac{n}{(n+2)(n+1)} a_n$ $a_{n+2} = \frac{1}{(n+2)(n+1)} a_{n+1} - \frac{1}{(n+2)(n+1)} a_n$ $a_{n+2} = \frac{n}{(n+2)(n+1)} a_{n+1} - \frac{1}{n+1} a_n$

Determine a lower bound for the radius of convergence of a power series solution centered at $x = 2$ of the differential equation

$$(x+3)^2 y'' + xy' - y = 0$$

5 4 3 2 1

Find a power series solution to the initial value problem

$$y'' + xy' = 0 \qquad y(0) = 0, \quad y'(0) = 1$$

$y = x - \frac{1}{6}x^3 + \frac{1}{40}x^5 - \frac{1}{336}x^7 + \ldots$ $y = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \ldots$ $y = x + \frac{1}{6}x^2 + \frac{1}{20}x^3 + -\frac{1}{35}x^4 + \ldots$ $y = x - \frac{1}{4}x^2 + \frac{1}{75}x^3 - \frac{1}{116}x^5 + \ldots$ $y = x - \frac{1}{12}x^3 + \frac{1}{105}x^5 - \frac{1}{286}x^7 + \ldots$