Math 226: Calculus IV Name:________________

Final Exam *May 6, 1991* Score:________________

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 6 points each.

Compute the matrix product

$$0 - 123410 - 20610112 - 2 - 3344$$

$420 - 2 - 69 - 9$ $0 - 2 - 6124 - 20 - 8012 - 30$ $0 - 18 - 2018$ $0 - 1238 - 204018 - 30$ *not defined*

Solve the following system of linear equations.

$$\begin{array}{ccccccccccc} x_1 & + & 2x_2 & + & x_3 & + & 2x_4 & + & x_5 & = & -4 \\ 2x_1 & + & x_2 & + & 2x_3 & + & x_4 & + & 2x_5 & = & 1 \\ 3x_1 & + & 2x_2 & + & 3x_3 & + & 2x_4 & + & 3x_5 & = & 0 \\ 4x_1 & + & 5x_2 & + & 4x_3 & + & 5x_4 & + & 4x_5 & = & -7 \end{array}$$

$x_1 = 2 - x_3 - x_5$ $x_1 = 2$ $x_1 = -4 - x_3$ $x_1 = 0$ *no solution*
$x_2 = -3 - x_4$ $x_2 = -3$ $x_2 = -3 - x_4$ $x_2 = -4$
x_3, x_4, x_5 free $x_3 = x_4 = x_5 = 0$ $x_5 = 0$ $x_3 = x_4 = 1$
x_3, x_4 free $x_5 = 0$

Compute the inverse of the matrix

$$001111100$$

The (3,2)-entry of the inverse is: 0 −1 1 1/2 −1/2

Compute the determinant of

$$1020030450600708$$

16 −20 24 −12 0

Find a maximal linearly independent subset of the vectors

$$X_1 = (1,0,1,1,0,0)$$
$$X_2 = (1,1,0,0,1,1)$$
$$X_3 = (1,1,1,1,1,1)$$
$$X_4 = (0,0,1,1,0,0)$$
$$X_5 = (0,1,0,0,1,1)$$

X_1, X_2, X_3 X_1, X_2, X_3, X_4 X_2, X_3, X_4 X_1, X_3, X_4, X_5 X_1, X_3, X_5

Find the general solution to

$$y' + y = e^x$$

$y = \frac{1}{2}e^x + ce^{-x}$ $y = \frac{1}{2} + ce^{-x}$ $y = 2e^x$ $y = e^{2x} + ce^x$ $y = \frac{1}{2}e^{2x} + ce^{-x}$

Solve the initial value problem

$$\frac{dy}{dx} = \frac{xy}{1+x^2}, \qquad y(0) = 1$$

$y = \sqrt{1+x^2}$ $y = 1 + \ln(\sqrt{1+x^2})$ $y = e^{\tan^{-1}(x)}$ $y = 1 + \ln(1 + \tan^{-1}(x))$ $y = 0$

Ground water seeps into a cylindrical retention pond at a rate of 100 ft^3 per minute. There is storm drain above the pond and when the water reaches this level it drains out at a rate proportional to the height h of the pond *above* the storm drain. A county engineer measures the flow in the storm drain to be 50 ft^3 per minute when h is 3 in. Find the limiting value for h. 6 in 1 ft 4 in 2 ft 8 in

Suppose the population N of spotted owls in a certain national park satisfies a logistic equation of the form

$$\frac{dN}{dt} = \left(\frac{500}{N} - 1\right)\left(1 - \frac{1000}{N}\right)N^3$$

If the current population of spotted owls in the park is 100, what is the long term population predicted by this model. 0 100 500 1000 *no limit*

A body of mass m is projected upward with initial veliocity v_0 in a medium offering resistance equal to the magnitude of the velocity. Find an expression for the time t when the body reaches its maximum height. $t = m\ln(\frac{v_0}{mg}+1)$ $t = m\ln(\frac{v_0}{mg})$ $t = \frac{v_0}{g}$ $t = \frac{v_0}{mg}$ $t = \sqrt{\frac{mv_0}{g}}$

Find the unique solution to

$$(3x^2y + 2y)\,dx + (x^3 - 2y + 2x)\,dy = 0$$

satisfying $y(1) = 1$. $x^3y - y^2 + 2xy = 2$ $x^3y + y^2 = 2$ $x^3y + 2xy = 3$ $3x^2y + 2xy + y^2 = 6$ $x^3 - y^2 = 0$

Find an integrating factor $\mu(x)$ for the equation

$$(y + (y - 2x)e^{-x})\,dx + (1 + xe^{-x})\,dy = 0$$

$\mu(x) = e^x$ $\mu(x) = 1$ $\mu(x) = x$ $\mu(x) = xe^x$ $\mu(x) = e^{2x}$

Solve the equation

$$\frac{dy}{dx} = \frac{xy + y^2}{x^2}$$

by making a substitution $v = \frac{y}{x}$. $y = \frac{x}{c - \ln(x)}$ $y = x(\ln(x) + c)$ $y = \frac{c}{x^2}$ $y = -\frac{1}{\ln(x)} + c$ $y = \frac{x}{c - x^2}$

Compute the Wronskian of the functions

$$y_1 = x\cos(x) \qquad y_2 = x\sin(x)$$

x^2 $x^2(1 - 2\sin^2(x))$ $x^2(1 - 2\cos^2(x))$ $2x\sin(x)\cos(x)$ 0

Solve the initial value problem

$$y'' + y(y')^3 = 0$$
$$y(0) = 1, \quad y'(0) = 2$$

$y = (6x + 1)^{1/3}$ $y = (4x + 1)^{1/2}$ $y = y = e^{2x}$ $y = (1 - 4x)^{-1/2}$ $y = (1 - 6x)^{-1/3}$

Find the general solution of

$$6y'' - 7y' + 2y = 0$$

$y = c_1e^{x/2} + c_2e^{2x/3}$ $y = c_1e^{3x/2} + c_2e^{x/3}$ $y = c_1e^{-x} + c_2e^{2x/3}$ $y = c_1e^{2x} + c_2e^{-2x/3}$ $y = c_1e^{-2x/3} + c_2e^{x/3}$

Find the solution of

$$y'' - 4y' + 4y = 0$$

satisfying $y(0) = 0$ and $y'(0) = 2$. $y = 2xe^{2x}$ $y = \frac{1}{2}(e^{2x} - e^{-2x})$ $y = \sin(2x)$ $y = \cos(2x)$ $y = x(e^{2x} + e^{-2x})$

A mass weighing 2 lbs stretches a spring 6 in. The mass is pushed upward, contracting the spring a distance of 1 in, and then released. Suppose air resistance exerts a damping force given by $0.5u'(t)$ where $u(t)$ is the displacement (in ft) from equilibrium with the positive direction down. Determine the subsequent motion. ($g = 32$ ft/sec^2) $u = -\frac{1}{12}e^{-4t}(\cos(4\sqrt{3}t) + \frac{1}{\sqrt{3}}\sin(4\sqrt{3}t))$ $u = -\frac{1}{12}e^{-t/8}(\cos(\frac{\sqrt{107}}{8}t) + \frac{1}{\sqrt{107}}\sin(\frac{\sqrt{107}}{8}t))$ $u = -\frac{1}{12}e^{-2t}(\cos(\frac{\sqrt{15}}{2}t) + \frac{4}{\sqrt{15}}\sin(\frac{\sqrt{15}}{2}t))$ $u = -\frac{1}{12}e^{-8t}(\cos(\frac{\sqrt{63}}{4}t) + \frac{32}{\sqrt{63}}\sin(\frac{\sqrt{63}}{4}t))$ $u = -\frac{1}{12}e^{-t/4}(\cos(\frac{\sqrt{83}}{6}t) + \frac{3}{2\sqrt{83}}\sin(\frac{\sqrt{83}}{6}t))$

Find the form of a particular solution of

$$y'' - 4y' + 3y = e^x + \cos(x)$$

$y_p = Axe^x + B\cos(x) + C\sin(x)$ $y_p = Ae^x + B\cos(x) + C\sin(x)$ $y_p = Ax^2e^x + x(B\cos(x) + C\sin(x))$ $y_p = (Ax^2 + Bx + C)e^x + x(D\cos(x) + E\sin(x))$ $y_p = Ae^x + x^2(B\cos(x) + C\sin(x))$

Given that $y_1 = x$ and $y_2 = x^{-1}$ are solutions to a second order homogeneous linear equation

$$L[y] = 0, \qquad (x > 0)$$

find a particular solution y_p to the equation

$$L[y] = x^3, \qquad (x > 0)$$

$y_p = \frac{1}{24}x^5$ $y_p = \frac{1}{4}x^3 - x^2 + 1$ $y_p = \frac{1}{12}x^5 - \frac{1}{8}x^3$ $y_p = \frac{1}{8}x^3$ $y_p = \frac{1}{3}x^{-3} - \frac{1}{2}x^3$

Classify the singular points of the equation

$$x(x^2-1)y'' + x(1-x)y' + y = 0$$

$x = 0$, $x = \pm 1$ regular $x = 0$ regular, $x = \pm 1$ irregular $x = 0$, $x = 1$ regular; $x = -1$ irregular $x = 0$ irregular, $x = \pm 1$ regular $x = 0$, $x = -1$ regular, $x = 1$ irregular

Compute the power series expansion of $f(x) = \frac{1}{1-x}$ centered at $x_0 = -1$. $\sum_{n=0}^{\infty} \frac{1}{2^{n+1}}(x+1)^n$ $\sum_{n=0}^{\infty} \frac{1}{2^n}(x-1)^n$ $\sum_{n=0}^{\infty} \frac{1}{n!}(x+1)^n$ $\sum_{n=0}^{\infty} x^n$ $\sum_{n=0}^{\infty} \frac{1}{n}(x-1)^n$

Determine the recurrence relation for the coefficients of a power series solution centered at $x_0 = 0$ of the equation

$$y'' + x^2 y = 0$$

$a_n = -\frac{1}{n(n-1)}a_{n-4}$ $a_{n+1} = -\frac{1}{(n+1)n}a_n$ $a_n = -\frac{1}{n(n+1)}a_{n-2}$ $a_{n+1} = -\frac{1}{(n+1)}a_{n-2} + \frac{1}{n(n-1)}a_{n-4}$ $a_{n+2} = -\frac{1}{(n+1)}a_{n-2}$

Find the general solution to

$$x^2y'' - 5xy' + 13y = 0, \qquad (x > 0)$$

$y = x^3(c_1\cos(2\ln(x)) + c_2\sin(2\ln(x)))$ $y = x^{-3}(c_1\cos(\ln(x)) + c_2\sin(\ln(x)))$ $y = c_1x^{-3} + c_2x^2$ $y = e^{-5x/2}(c_1\cos(\sqrt{17}x) + c_2\sin(\sqrt{17}x))$ $y = c_1x^{5x/2} + c_2x^{5x/2}\ln(x)$

The equation

$$x^2y'' + x(1+x)y' - \frac{1}{4}y = 0$$

has a regular singular point at $x_0 = 0$. Find a series solution corresponding to the larger of the exponents of the singularity. $y = x^{1/2}(1 - \frac{1}{4}x + \frac{1}{16}x^2 - \frac{5}{384}x^3 + \ldots)$ $y = x^{-1/2}(1 - x + \frac{1}{4}x^2 - \frac{1}{120}x^3 + \ldots)$ $y = x^{-(1+\sqrt{2})/2}(1 - \frac{1}{2}x + \frac{1}{4}x^2 - \frac{3}{196}x^3 + \ldots)$ $y = x^{(1+\sqrt{2})/2}(1 - x + \frac{1}{12}x^2 - \frac{1}{360}x^3 + \ldots)$ $y = x^{3/2}(1 - x + \frac{1}{15}x^2 - \frac{13}{240}x^3 + \ldots)$