Math 226: Calculus IV Name:________________________

Exam II *March 19, 1992* Score:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Find the general solution of the differential equation $y' + 2xy = x$. $y = \frac{1}{2}(ce^{-x^2} + 1)$ $y = \frac{1}{2}(c + e^{x^2})$ $y = c + x^2$ $y = \frac{1}{2}(c + e^{2x})$ $y = ce^{-2x} + 1$

Solve the initial value problem $y' - xy^2 = 0, \quad y(0) = 3$. $y = \frac{6}{2-3x^2}$ $y = (3\ln(x))^{1/3} + 3$ $y = \frac{3}{x^2+1}$ $y = \dfrac{3}{\ln(x+1)}$ $y = \frac{1}{2}x^2 + 3$

Solve the differential equation $(3x^2y - y^2 - 2x)\,dx + (x^3 - 2xy + 1)\,dy = 0$. $xy^2 - (x^3+1)y + x^2 = c$ $x^3y^2 - \frac{1}{3}y^3 - x^2 + \frac{4}{3} = c$ $f(x,y) = x^3y - xy^2 - x^2$ $f(x,y) = x^3y - xy^2 + y$ $y = \frac{1}{2}[(x^3+1) + \sqrt{(x^3+1)^2 - x^2 + c}]$

Find the integral curve of the differential equation

$$\frac{dy}{dx} = \frac{x(y^2+1)}{1-x^2y}$$

that passes through the point (1,0). $x^2y^2 - 2y + x^2 = 1$ $x^2y^2 + x^2 = 1$ $\frac{1}{2}x^2y^2 - y = 0$ $y = \frac{\sqrt{2-2x^2}}{x}$ $y = \frac{3x^2(y^2+1)}{6-2x^3y} - \frac{1}{2}$

Solve the differential equation

$$\frac{dy}{dx} = \frac{y(x^2-y^2)}{x^3}$$

subject to the initial condition $y(1) = -\frac{1}{2}$. $y = -\frac{x}{\sqrt{2\ln(x)+4}}$ $\frac{2}{3}x^3y - \frac{1}{3}y^3 = -\frac{1}{2}$ $y = \frac{4y(x^3-3xy^2)}{x^4}$ $\frac{1}{3}x^3y - \frac{1}{4}y^4$ $y = \frac{x}{2\ln(x)+2}$

Two populations of bacteria, type A and type B, are increasing in a lab experiment at rates proportional to their current sizes. The type A bacteria double their numbers every week and the type B bacteria triple their numbers every week. If there are twice as many type A bacteria as type B initially, determine how long it will take for the number of type B bacteria to equal the number of type A bacteria. 11.97 days 13.51 days 9.33 days 10.15 days 8.67 days

Suppose a population N grows according to the law

$$\frac{dN}{dt} = \alpha(N^2 - 10N + 24)(N^2 - 5N + 6)$$

where α is a positive constant. If $N(0) = 5$, determine the limiting value for $N(t)$ as $t \to \infty$. 4 2 3 6 5

A solution of orange dye is being mixed in a 100 liter tank. The tank starts with 100 ℓ of pure water to which two streams of dye solution are being added. The red stream contains 1 g/ℓ of dye and the yellow stream contains 2 g/ℓ of dye. Both streams are flowing in at a rate of 1 ℓ/sec. The mixture is well-stirred and siphoned off so that the tank remains at 100 ℓ. Find an expression for the concentration $c(t)$ of dye in the orange solution leaving the tank. $c(t) = 1.5(1 - e^{-t/50})$ $c(t) = 0.5(1 + e^{-t/100})$ $c(t) = 100(1 - e^{-2t})$ $c(t) = 50(1 + e^{-2t})$ $c(t) = \frac{2}{3}(1 - \frac{1}{2}e^{-t/50})$

An object weighing 1 lb falls from rest in a medium offering resistance numerically equal to the square of the velocity. Find the velocity of the object as a function of t. (Assume $g = 32$ ft/sec^2.) $v = \frac{1-e^{-64t}}{1+e^{-64t}}$ ft/sec $v = (1 - e^{-32t})$ ft/sec $v = -16t^2$ ft/sec $v = \frac{1+e^{-32t}}{1-e^{-32t}}$ ft/sec $v = (1 + e^{-64t})$ ft/sec

Suppose a glass of ice-cold lemonade ($32°F$) is placed outside on a hot day ($90°F$) and warms up at a rate proportional to the difference between its temperature and the surrounding air. Ten minutes later the lemonade is $50°F$. How long will it take before the lemonade reaches $70°F$. 28.65 mins 25.35 mins 23.55 mins 30.45 mins 20.75 mins

A bank loans money at an annual rate of r compounded continuously. Suppose P dollars are borrowed and paid back continuously over N years. Find a formula for the amount k that must be paid back annually. $k = \frac{rP}{1-e^{-rN}}$ $k = \frac{Pe^{rN}}{N}$ $k = \frac{rP}{N}$ $k = \frac{rP}{e^{rN}-1}$ $k = \frac{P(1+e^{rN})}{N}$

Find an integrating factor for the differential equation

$$2x\,dx + (x^2 + \cos(y))\,dy = 0$$

e^y e^{-x} $\sin(y)$ $\frac{1}{x}$ e^x

Convert the second order equation

$$yy'' - (y')^3 = 0$$

into a first order equation by making the substitution $v = y'$. $\frac{dv}{dy} = \frac{v^2}{y}$ $yv' - v^3 = 0$ $v' = \frac{v}{y}$ $\frac{dv}{dx} = \frac{v^3}{x}$ $\frac{dv}{dy} - \frac{1}{y}v = 0$

Solve the initial value problem

$$x^3 y'' = (y')^2, \quad y(1) = 1, \quad y'(1) = 2$$

by making an appropriate substitution. $y = \frac{1}{3}(2x^3 + 1)$ $y = \frac{1}{2x^2 - 1}$ $y = \frac{x^2}{2x^2 - 1}$ $y = x^2$ $y = x + \frac{1}{2}\ln(2x - 1)$

Determine the interval on which one can expect the solution to the following initial value problem to be valid.

$$y'' = \frac{e^x - xy}{x^2 - 1}, \quad y(2) = 0, \quad y'(2) = 1$$

$(1, \infty)$ $(-1, 1)$ $(-\infty, -1)$ $(-\infty, \infty)$ *cannot be determined*