Math 226: Calculus IV Name:____________________

Exam III *April 21, 1992* Score:____________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Let $L[y] = y'' + p(x)y' + q(x)y$. Determine which of the following statements is FALSE. If y_1 and y_2 are linearly independent solutions to $y'' = f(x, y, y')$ then any solution to this differential equation can be written as $c_1y_1 + c_2y_2$ for some choice of constants c_1 and c_2. If y_1 and y_2 are linearly independent solutions to $L[y] = 0$ then any solution to this differential equation can be written as $c_1y_1 + c_2y_2$ for some choice of constants c_1 and c_2. y_1 and y_2 form a fundamental set of solutions to $L[y] = 0$ if and only if $L[y_1] = 0$, $L[y_2] = 0$, and $W(y_1, y_2) \neq 0$. y_1 and y_2 are linearly independent functions of x if and only if $W(y_1, y_2) \neq 0$. If y_1 and y_2 are solutions to $L[y] = 0$ then $c_1y_1 + c_2y_2$ is also a solution for any choice of constants c_1 and c_2.

Compute the Wronskian of the functions $f(x) = \cos(x^2)$ and $g(x) = \sin(x^2)$. $2x$ $4x$ $2x(\cos(x^2)+\sin(x^2))$ $2x(\sin(x^2) - \cos(x^2))$ 1

Given that $y_1 = x^{-1}$ is a solution of

$$x^2y'' + 3xy' + y = 0 \qquad (x > 0)$$

a second linearly independent solution can be found of the form $y_2 = vy_1$ where v is a non-constant function of x. Determine the differential equation v must satisfy. $xv'' + v' = 0$ $v' = x$ $v'' - \frac{1}{x}v' = 0$ $v'' + xv' = 0$ $v' = x^{-2}$

Find the general solution of the homogeneous equation $y'' + 2y' + 4y = 0$. $y = e^{-x}(c_1\cos(\sqrt{3}x) + c_2\sin(\sqrt{3}x))$ $y = e^{x}(3\cos(x) + 3\sin(x))$ $y = e^{-x}(c_1\cos(x) + c_2\sin(x))$ $y = e^{-2x}(c_1\cos(3x) + c_2\sin(3x))$ $y = e^{\sqrt{3}x}(c_1\cos(2x) + c_2\sin(2x))$

Suppose y satisfies $4y'' - 4y' + y = 0$, $y(0) = 0$, and $y'(0) = 1$. Find $y(1)$. $\sqrt{e}$ e $1/e$ $1/\sqrt{e}$ e^2

Solve the initial value problem

$$3y'' + 2y' - y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$y = (3e^{x/3} + e^{-x})/4$ $y = (\sqrt{3}e^{\sqrt{3}x} - 3e^{x})/(\sqrt{3} - 3)$ $y = (e^{x} + e^{-x})/2$ $y = (\sqrt{2}e^{\sqrt{2}x} + 2e^{-x})/(\sqrt{2} - 2)$ $y = (e^{-x} + 2e^{x/2})/3$

Find a particular solution, y_p to the differential equation

$$y'' - 2y' + 2y = xe^{x}$$

$y_p = xe^{x}$ $y_p = (x+1)e^{x}$ $y_p = (x^2+x)e^{x}$ $y_p = x^2e^{x}$ $y_p = xe^{x}\cos(x)$

Determine the form of a particular solution to the differential equation

$$y'' - 2y' = x\cos(x) + e^{2x}$$

$y_p = (a_0x + a_1)\cos(x) + (b_0x + b_1)\sin(x) + Axe^{2x}$ $y_p = (a_0x^2 + a_1x)\cos(x) + (b_0x^2 + b_1x)\sin(x) + Ae^{2x}$ $y_p = (a_0x^2 + a_1x)\cos(x) + (b_0x^2 + b_1x)\sin(x) + (Ax + B)e^{2x}$ $y_p = (a_0x^2 + a_1x)\cos(x) + Axe^{2x}$ $y_p = (a_0x + a_1)e^{2x}(b_1\cos(x) + b_2\sin(x)) + Ae^{2x}$

Let $y_p = u_1(x)x + u_2(x)e^{x}$ be a particular solution of the differential equation

$$(1-x)y'' + xy' - y = (x-1)^2 \qquad (x > 1)$$

given by the method of variation of parameters. Find u_1. $u_1(x) = x$ $u_1(x) = (x-1)^3/3$ $u_1(x) = (x-1)e^{x}$ $u_1(x) = x(x-1)$ $u_1(x) = -(x-1)^2/2$

A mass weighing 16 lbs stretches a spring 4 in. The mass is attached to a viscous damper with damping constant c. Determine the value of c if the spring-mass system is to be critically damped (i.e., damped just enough to eliminate oscillations). $c = 4\sqrt{6}$ $c = 4$ $c = 4\sqrt{3}$ $c = 8\sqrt{2}$ $c = 8$

Find the power series expansion for $f(x) = \frac{1}{x^2}$ centered at $x_0 = 1$. $f(x) = 1 - 2(x-1) + 3(x-1)^2 - 4(x-1)^3 + \cdots$ $f(x) = 1 + 2(x-1) + 6(x-1)^2 + 24(x-1)^3 + \cdots$ $f(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \cdots$ $f(x) = 1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x + \cdots$ $f(x) = 1 - x^2 + x^3 - x^4 + \cdots$

Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{3^n n^3}{n!} x^n$. ∞ $1/3$ 1 3 $\sqrt{3}$

Determine a power series solution to the initial value problem

$$y'' - x^2 y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

$y = 1 + \frac{1}{4\cdot 3}x^4 + \frac{1}{8\cdot 7\cdot 4\cdot 3}x^8 + \cdots$ $y = 1 + \frac{1}{3\cdot 2}x^3 + \frac{1}{6\cdot 5\cdot 3\cdot 2}x^6 + \cdots$ $y = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \cdots$ $y = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots$ $y = 1 - \frac{1}{4\cdot 3}x^4 + \frac{1}{8\cdot 7}x^8 - \cdots$

Determine the recurrence relation for the coefficients of a power series solution centered at $x_0 = 0$ of the differential equation

$$(1 - x^2)y'' + xy' - y = 0$$

$a_{n+2} = \frac{(n-1)^2}{(n+2)(n+1)}a_n$ $a_{n+2} = \frac{n}{(n+2)(n+1)}a_{n+1} - \frac{1}{(n+1)}a_n$ $a_{n+2} = \frac{n(n-1)}{(n+2)(n+1)}a_n$ $a_{n+2} = \frac{n}{(n+2)}a_{n+1} - \frac{1}{(n+2)(n+1)}a_n$ $a_{n+2} = \frac{(n-1)}{(n+2)(n+1)}a_n$

Determine a minimum for the radius of convergence of a power series solution centered at $x_0 = 3$ to the differential equation

$$(x^2 + x)y'' - xy' + (x+1)y = 0$$

3 1 2 ∞ 0