Math 226: Calculus IV Name:________________

Final Exam *May 5, 1992* Score:________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 6 points each.

Let $A = [1, 2, 3]$ and $B = 456$. Compute the *matrix product* $B \cdot A$.
48125101561218 [32] 123456 45681012121518 *not defined*

Let $A = 0 - 1111 - 1210$. The $(3, 2)$-entry of the inverse matrix A^{-1} is: -2 2 1 -1 0

Calculate the determinant of the matrix $13 - 142101002 - 10001$. -10 2 -4 -6 8

Find the general solution to the following system of linear equations:

$$\begin{aligned} x_1 + x_3 &= 1 \\ 2x_1 + x_2 + 4x_3 - x_4 &= 5 \\ 3x_1 + x_2 + 6x_3 &= 10 \\ 4x_1 + x_2 + 6x_3 - x_4 &= 7 \end{aligned}$$

$x_1 = -3 + x_4$ $x_1 = 1 - x_3$ $x_1 = -3$ $x_1 = -\frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_4$ *the system is inconsistent*
$x_2 = -5 + 3x_4$ $x_2 = 5 - 2x_3$ $x_2 = -5$ $x_2 = \frac{3}{2} - \frac{1}{2}x_2 - \frac{1}{2}x_4$
$x_3 = 4 - x_4$ $x_4 = -7$ $x_3 = 4$
$x_4 = 0$

If the Gram-Schmidt process is applied to the vectors

$$X_1 = [1, 1, 0, 1], \qquad X_2 = [0, 0, 1, 0], \qquad X_3 = [1, 0, 1, 1]$$

to obtain the vectors Q_1, Q_2, Q_3, then: $Q_3 = \frac{1}{\sqrt{6}}[1, -2, 0, 1]$ $Q_3 = \frac{1}{\sqrt{2}}[1, -1, 0, 0]$ $Q_3 = \frac{1}{\sqrt{2}}[0, 1, 0, -1]$ $Q_3 = \frac{1}{\sqrt{14}}[2, -3, 0, 1]$ $Q_3 = \frac{1}{\sqrt{6}}[1, 1, 0, -2]$

Suppose y satisfies $xy' - 3y = 4x$ and $y(1) = 0$. Calculate $y(2)$. 12 4 8 6 10

Determine which of the following equations defines the integral curves of the differential equation

$$x^2 y' + y^2 = 1$$

$y = \frac{c - e^{2/x}}{c + e^{2/x}}$ $y = \frac{c - e^{2x}}{c + e^{2x}}$ $y = \frac{e^{x/2} - c}{e^{x/2} + c}$ $y = \frac{e^x - c}{e^x + c}$ $y = \frac{e^{-x} - c}{ce^x + 1}$

A 50 gal drum is filled with water having a 10% concentration of cleaning fluid. Fresh water is added to the drum at a rate of 2 gal/min and the well-stirred mixture is siphoned out of the drum at the same rate. Calculate how long it will take the concentration of cleaning fluid to fall below 1%. 57.6 min 45.3 min 37.5 min 69.2 min 28.4 min

Classify the equilibrium solutions of the logistic equation

$$\frac{dN}{dt} = N(1 - N^2)$$

$N = 1$ stable, $N = 0$ unstable, $N = -1$ stable $N = 1$ unstable, $N = 0$ stable, $N = -1$ stable $N = 1$ stable, $N = 0$ unstable, $N = -1$ unstable $N = 1$ unstable, $N = 0$ stable, $N = -1$ unstable $N = 1$ stable, $N = 0$ stable, $N = -1$ unstable

Air resistance on a 160 lb skydiver with an unopened parachute is proportional to the square of the velocity, kv^2. After falling 20 seconds, the skydiver is observed to have a steady rate of descent of 200 ft/sec. What value of k yields this terminal velocity? (Use the given units of lbs, ft, sec.) 0.004 0.16 0.02 0.8 5.0

Find the equation of the integral curve for the differential equation

$$(e^x \cos(y) + 2x - y^2)\, dx - (e^x \sin(y) + 2xy - 1)\, dy = 0$$

that passes through the point $(0, 0)$.

$e^x\cos(y)+x(x-y^2)+y=1$ $e^x\sin(y)+xy^2-y=0$ $e^x\cos(y)+x^2-xy^2=1$ $e^x\sin(y)+xy^2-y=0$ $e^x(\cos(y)-\sin(y))-x^2y+y=1$

Find an integrating factor μ for the differential equation

$$(e^x+x^2e^{y^2})\,dx-2y(e^x+e^{y^2})\,dy=0$$

$\mu=e^{-y^2}$ $\mu=e^{-x}$ $\mu=e^{y^2}$ $\mu=e^x$ $\mu=\frac{1}{2}y$

Solve the differential equation

$$\frac{dy}{dx}=\frac{x\tan(y/x)+y}{x}$$

$\sin(y/x)=cx$ $\ln|\sin(y/x)|=\frac{1}{2}x^2+c$ $\sec^2(y/x)=\frac{1}{2}x^2+c$ $\cos(y/x)=cy$ $\ln|\cos(y/x)|=y+c$

Solve the initial value problem

$$y^2y''-(y')^3=0,\quad y(0)=1,\quad y'(0)=2$$

$x=\frac{1}{2}(1-y)+\ln|y|$ $y=e^{2x}$ $y=\sqrt{4x+1}$ $y=-2+\sqrt{4x+9}$ $x=\frac{1}{2}(1-y)^2$

Given that $y_1=e^x$ is a solution to the differential equation

$$(x-1)y''-xy'+y=0$$

A second linearly independent solution can be found of the form $y_2=vy_1$ where v is a non-constant function of x. Determine the differential equation v must satisfy. $(x-1)v''+(x-2)v'=0$ $(x-1)v''-xv'+2v=0$ $(x-1)v''+(x+1)v'=0$ $e^x(x-1)v''-xv'=0$ $(x-1)v''-xv'+2e^xv=0$

Determine a suitable form for a particluar solution y_p of the differential equation

$$y''-4y'+4y=4xe^{2x}+x\sin(2x)$$

$y_p=x^2(A_1x+A_0)e^{2x}+(B_1x+B_0)\sin(2x)+(C_1x+C_0)\cos(2x)$ $y_p=x(A_1x+A_0)e^{2x}+x(B_1x+B_0)\sin(2x)+x(C_1x+C_0)\cos(2x)$ $y_p=(A_1x+A_0)e^{2x}+(B_1x+B_0)\sin(2x)+(C_1x+C_0)\cos(2x)$ $y_p=Axe^{2x}+Bx\sin(2x)+Cx\cos(2x)$ $y_p=Ax^2e^{2x}+(B_1x+B_0)\sin(2x)+(C_1x+C_0)\cos(2x)$

Let $L[y]=y''+p(x)y'+q(x)$. Suppose x and x^{-1} are solutions to $L[y]=0$. Calculate a particular solution $y_p=u_1(x)x+u_2(x)x^{-1}$ to the differential equation $L[y]=x^3$ using the method of variation of parameters. $y_p=\frac{1}{24}x^5$ $y_p=\frac{1}{8}x^4-\frac{1}{12}x^6$ $y_p=\frac{1}{6}x^6$ $y_p=\frac{1}{2}(x^4-x)$ $y_p=\frac{1}{20}x^5-\frac{1}{6}x^3$

Suppose a man suspended from a bungee cord stretches the cord 16 feet longer than its natural length of 34 ft. The man jumps from a bridge while attached to this cord and arrives at a distance of 50 ft below the bridge with a downward velocity of 75 ft/sec. At this point the cord starts acting like a spring (i.e., it obeys Hooke's Law.) Assuming there are no frictional forces, how long will it take for the man to return to the point 50 ft below the bridge? $\pi\sqrt{2}/2$ $\pi\sqrt{6}/2$ $\pi/2$ $\pi\sqrt{3}/3$ $\pi\sqrt{6}/3$

Find the recurrence relation for the coefficients of a power series solution to the differential equation

$$(1+x^2)y''-xy'+y=0$$

$a_n=\frac{(n-3)^2}{n(n-1)}a_{n-2}$ $a_{n+2}=\frac{n}{(n+2)(n+1)}a_n$ $a_{n+2}=-\frac{1}{(n+2)(n+1)}a_n$ $a_n=\frac{n-1}{n}a_{n-2}$ $a_n=\frac{(n-1)^2}{(n+2)(n+1)}a_{n-2}$

Find the first three non-zero terms of a power series solution to the initial value problem

$$(4-x^2)y''+2y=0,\quad y(0)=0,\quad y'(0)=1$$

$y=x-\frac{1}{12}x^3-\frac{1}{240}x^5-\cdots$ $y=x-\frac{1}{24}x^3+\frac{1}{20}x^5-\cdots$ $y=x-\frac{1}{24}x^3-\frac{1}{480}x^5-\cdots$ $y=x-\frac{1}{12}x^3+\frac{1}{120}x^5-\cdots$ $y=x-\frac{1}{6}x^3-\frac{1}{80}x^5-\cdots$

Classify the singular points of the differential equation

$$x^2(x-1)^2y''+2xy'+(x^2-1)y=0$$

$x=0$ regular, $x=1$ irregular $x=0$ regular, $x=1$ regular, $x=-1$ irregular $x=0$ regular, $x=1$ regular $x=0$ irregular, $x=1$ irregular $x=0$ irregular, $x=1$ regular, $x=-1$ regular

Determine the general solution of the differential equation

$$x^2y'' - 5xy' + 10y = 0$$

$y = x^3(c_1 \cos(\ln|x|) + c_2 \sin(\ln|x|))$ $y = x^{5/2}(c_1 \cos(\frac{\sqrt{15}}{2} \ln|x|) + c_2 \sin(\frac{\sqrt{15}}{2} \ln|x|))$ $y = e^{5/2}(c_1 \cos(\frac{\sqrt{15}}{2}x) + c_2 \sin(\frac{\sqrt{15}}{2}x))$ $y = e^{3x}(c_1 \cos(x) + c_2 \sin(x))$ $y = c_1x^{-3} + c_2x^{-5}$

The equation

$$x^2y'' + (x + \sin(x))y' - y = 0$$

has a regular singular point at $x = 0$. Determine the exponents of the this singularity. $r_1 = \frac{-1+\sqrt{5}}{2}, \quad r_2 = \frac{-1-\sqrt{5}}{2}$ $r_1 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad r_2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$ $r_1 = 1, \quad r_2 = -1$ $r_1 = 1, \quad r_2 = 0$ $r_1 = r_2 = 1$

The differential equation

$$2x^2y'' + 3xy' + (2x^2 - 1)y = 0$$

has a regular singular point at $x = 0$. Find a series solution corresponding to the larger of the exponents of the singularity. $y = x^{1/2}\left(1 - \frac{1}{7}x^2 + \frac{1}{154}x^4 - \cdots\right)$ $y = x^{1/2}\left(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 - \cdots\right)$ $y = x\left(1 - x^2 + \frac{1}{10}x^4 - \cdots\right)$ $y = x^{-1}\left(1 - \frac{1}{14}x^2 + \frac{1}{182}x^4 - \cdots\right)$ $y = x\left(1 - \frac{1}{10}x^2 + \frac{1}{440}x^4 - \cdots\right)$

Solve the initial value problem

$$x^2y'' + 7xy' + 9y = 0, \qquad y(1) = 0, \qquad y'(1) = 2$$

$y = 2\ln|x|/x^3$ $y = (x^3 - x^{-3})/3$ $y = -\frac{2}{3}x^{-3}$ $y = \frac{2}{7}(x^{7/2} - x^{-7/2})$ $y = \frac{1}{\sqrt{13}}x^{7/2}\sin(\frac{\sqrt{13}}{2}\ln|x|)$