1. The general solution of the equation $xy' - y = x^3$ is

a. $\frac{x^3}{3} + Cx$ b. $\frac{x^3}{2} + Cx$ c. $x^2 + Cx$ d. $x^3 + \frac{C}{x}$ e. $\frac{x^3}{3} + \frac{C}{x}$

2. Consider the equation $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$. Which statement is true?

a. $y = x \tan(\log x + x)$ is a solution

b. $y = x \tan(\log x - x)$ is a solution

c. $y = x \tan(\log x)$ is a solution

d. None of the above is correct

e. All (a-c) are correct

3. Let y be the solution of $\frac{dy}{dx} = \frac{xy}{1 + x^2}$, $y(0) = 1$ Then

a. $y(1) = 1 + \log \sqrt{2}$ b. $y(1) = 0$ c. $y(1) = e^{\frac{\pi}{4}}$

d. $y(1) = 1 + \log \left(\frac{\pi}{4} + 1\right)$ e. $y(1) = \sqrt{2}$

4. Consider the differential equation with initial condition

$$y' = \sqrt{1 - y^2} \quad , \quad y(0) = \frac{1}{2}$$

a. There exists a unique solution for the initial value problem.

b. There exist two solutions for the initial value problem.

c. There exist infinitely many solutions for the initial value problem.

d. There exists no solution for the initial value problem.

e. There exist more than two but finitely many solutions.

5. Let y be the solution to the equation

$$(y \cos x + 2x\ e^y) \quad + \quad (\sin x + x^2 e^y - 1)\ y' = 0$$

Which is true?

a. $y \sin x - x^2\ e^y + y = C$

b. $y \sin x + x^2 e^y - y = C$

c. $x \sin y - y^2\ e^y + x = C$

d. $x \sin y + x^2 e^y + x = C$

e. $y \sin x + x^2\ e^y + y = C$

6. Consider the equation $\frac{dN}{dt} = N(N-1)(N-2)$ Which is true?

a. N=2 is the only stable critical solution

b. N=2 and N=0 are stable critical solutions

c. N=1 is the only stable critical solution

d. There is no stable critical solution

e. N=0 is not a critical solution

7. Suppose the half-life of a certain radioactive isotope is 20 days. If we start with 100 g of isotope, find the amount left after 10 days.

a. $\frac{100}{\sqrt{2}}$ b. $\frac{100}{\sqrt[3]{2}}$ c. $100\sqrt{2}$ d. $100\sqrt[3]{2}$ e. $\frac{100}{\sqrt{3}}$

8. A body with m=1 falls from rest in a medium offering resistance force equal to gv^2, where g is the gravitational constant and v is the velocity.

a. Write down the differential equation of the motion.

b. Find the relation between the velocity v and the time t.

c. Find the limiting velocity

9. Solve the equation $(3xy + y^2) + xy\,y' = 0$, $y(1) = 1$

Hint: Find an integrating factor which is a function of x.