1. The general solution of the equation $x^2\ y'' - 4\ xy' + 4y = 0$ is

a. $y = c_1\ x^2 + c_2\ x^2 \log x$

b. $y = c_1\ x + c_2\ x^4$

c. $y = c_1\ e^{2x} + c_2\ x\ e^{2x}$

d. $y = c_1\ x^2 + c_2\ x^3$

e. $y = c_1\ x^2 \cos(2 \log x) + c_2\ x^2 \sin(2 \log x)$

2. The radius of convergence of power series $\sum_{n=0}^{\bullet} \frac{(-1)^n}{2^n}\ x^n$ is

a. 1

b. 2

c. $\frac{1}{2}$

d. 4

e. $\frac{1}{4}$

3. Consider the equation $y'' + (\cos x)\ y=0$ with initial condition

$$y(0) = 0,\ y'(0) = 1$$

Then $y^{(3)}(0) =$

a. 0

b. – 2

c. 2

d. 1

e. – 1

4. The exponents of singularity for equation

$$x^2 y'' - x(2 + x) y' + (2 + x^2) y = 0$$

are

a. 1, 2

b. –1, –2

c. 1, –2

d. –1, 2

e. –1, –1

5. Find the first two non-zero terms of a power series solution to the intial value problem

$$(2 + x^2) y'' + 4y = 0,\quad y(0) = 0,\ y'(0) = 1.$$

a. $y = x - \frac{1}{3} x^3 + \ldots$

b. $y = x - 2x^3 + \ldots$

c. $y = x + \frac{1}{3}\ x^3\ + \ldots$

d. $y = x - \frac{1}{6}\ x^3\ + \ldots$

e. $y = x + \frac{1}{6}\ x^3\ + \ldots$

6. This power series solution is convergent at least in the interval.

a. $(-\infty, \infty)$

b. $(-1 - \sqrt{2}\ ,\ -1 + \sqrt{2}\ )$

c. $(1 - \sqrt{2}\ ,\ 1 + \sqrt{2}\ )$

d. $(-2, 2)$

e. $(-\sqrt{2}\ , \sqrt{2}\ )$

7. The Hermite equation is $y'' - 2\,x\,y' + \lambda\,y = 0$

Find the recurrence relation of the power series solution

$$y = \sum_{n=0}^{\bullet} a_n\,x^n$$

When $\lambda = 6$, find a polynomial solution.

8. $r = 1$ is an exponent of singularity for the equation

$$xy'' - y = 0$$

Write the general expression for coefficients of the series solution corresponding to the exponent. Find the radius of convergence.