Math 226 Final

1. The general solution of the equation $xy' - y = x^3$ is

a. $\frac{x^3}{3} + Cx$

b. $\frac{x^3}{2} + Cx$

c. $x^2 + Cx$

d. $x^3 + \frac{C}{x}$

e. $\frac{x^3}{3} + \frac{C}{x}$

2. Let y be the solution satisfying $\frac{dy}{dx} = -\frac{2x + 3y}{3x + 4y}$, $y(0) = 1$
Then y satisfies

a. $2y^2 - 3xy + x^2 = 2$

b. $2y^2 - 3xy - x^2 = 2$

c. $2y^2 + 3xy + x^2 = 2$

d. $2y^2 + 3xy - x^2 = 2$

e. $2y^2 + 6xy + x^2 = 2$

3. Solve the initial value problem $\frac{dy}{dx} = \frac{xy}{1 + x^2}$, $y(0) = 1$

a. $y = 1 + \ln\left(\sqrt{1 + x^2}\right)$

b. $y = 0$

c. $y = e^{\tan^{-1}(x)}$

d. $y = \sqrt{1 - x^2}$

e. $y = \sqrt{1 + x^2}$

4. The solution y of the differential equation $\frac{dy}{dx} = y(1 - y)$, $y(0) = \frac{1}{2}$ is

a. $\frac{e^x}{1 + e^x}$

b. $\frac{1 + e^x}{e^x}$

c. $\frac{e^{-x}}{1 + e^{-x}}$

d. $\frac{e^{-x}}{1 + e^x}$

e. $\frac{-e^{-x}}{1 - 2e^{-x}}$

5. Let y be the solution to the equation

$$(y \cos x + 2xe^{y}) + (\sin x + x^2 e^{y} - 1)\, y' = 0 .$$

Then y satisfies

a. $y \sin x + x^2 + y = C$

b. $y \sin x - x^2 e^{y} + y = C$

c. $y \sin x + x^2 e^{y} - y = C$

d. $x \sin y - y^2 e^{y} + x = C$

e. $x \sin y + x^2 e^{y} + x = C$

6. Compute the Wronskian of the functions

$$y_1 = e^{x} \cos 2x, \; y_2 = e^{x} \sin 2x$$

a. e^{x}

b. e^{2x}

c. $2e^{2x}$

d. $2e^{x}$

e. 0

7. The general solutions of the equation $y'' + 3y = 0$ are

a. $y = C_1 e^{-3x} + C_2 x$

b. $y = C_1 e^{3x} + C_2 x$

c. $y = C_1 e^{-3x} + C_2$

d. $y = C_1 e^{3x} + C_2$

e. $y = C_1 \cos \sqrt{3}\, x + C_2 \sin \sqrt{3}\, x$

8. Let y_p be a particular solution of the equation $y'' + 4y' + 5y = 3x + 2$. Then y_p could be

a. $\frac{3}{5} x + \frac{2}{25}$

b. $\frac{3}{5} x - \frac{2}{25}$

c. $\frac{3}{5} x - \frac{2}{5}$

d. $\frac{3}{5} x + \frac{2}{5}$

e. None of the above

9. A correct form of a particular solution of $y'' + 9y = \cos 3x$ should be

a. $A \sin 3x$

b. $B \cos 3x$

c. $A \sin 3x + B \cos 3x$

d. $A x \sin 3x + B x \cos 3x$

e. None of the above

10. Let u be the solution to the equation

$$u'' + 4u = 0 , \quad u(0) = 1 , \quad u'(0) = 1 .$$

Express the solution in $u = R \cos(wt - \delta)$ form. Then R =

a. $\frac{\sqrt{2}}{2}$

b. $\sqrt{2}$

c. $\frac{1}{2}$

d. $\frac{\sqrt{5}}{2}$

e. $\sqrt{5}$

11. The general solution of the equation $x^2 y'' - 4xy' + 4y = 0$ is

a. $y = c_1 x^2 + c_2 x^2 \log x$

b. $y = c_1 e^{2x} + c_2 x e^{2x}$

c. $y = c_1 x^2 + c_2 x^3$

d. $y = c_1 x + c_2 x^4$

e. $y = c_1 x^2 \cos(2 \log x) + c_2 x^2 \sin(2 \log x)$

12. The exponents of singularity for equation

$$x^2\ y'' - x\,(2 - x)\,y' + (2 - 2x^2)\,y = 0$$

are

a. –1, –2

b. 1, 2

c. –1, 2

d. 2, –1

e. –1, 1

13. Consider the equation $y'' - (\cos x)\,y = 0$ with initial condition

$$y\,(0) = 0,\ \ y'\,(0) = 1$$

Then $y^{(3)}\,(0) =$

a. 0

b. –2

c. 2

d. 1

e. –1

14. Find the first two non-zero terms of a power series solution to the inital value problem

$$(1 + x^2)\, y'' + 2y = 0\ , \quad y(0) = 0,\ \ y'(0) = 1$$

a. $y = x - \frac{1}{3}\, x^3 + \ldots$

b. $y = x + \frac{1}{3}\, x^3 + \ldots$

c. $y = x + \frac{1}{6}\, x^3 + \ldots$

d. $y = x - \frac{1}{6}\, x^3 + \ldots$

e. $y = x - 2\, x^3$

15. For equation $y'' - xy' + \lambda y = 0$, find the recurrence relation of the power series solution $y = \sum_{n=0}^{\infty} a_n x^n$.

a. $a_{n+2} = \dfrac{(n + \lambda)\, a_n}{(n + 2)\,(n + 1)}$

b. $a_{n+2} = \dfrac{(n - \lambda)\, a_n}{(n + 2)\,(n + 1)}$

c. $a_{n+2} = \dfrac{(2n - \lambda)\, a_n}{(n + 2)\,(n + 1)}$

d. $a_{n+2} = \dfrac{(2n + \lambda)\, a_n}{(n + 2)\,(n + 1)}$

e. None of the above

16. (continued) When $\lambda=3$, find a polynomial solution.

a. $x - \frac{1}{3}\, x^3$

b. $x + \frac{1}{3}\, x^3$

c. $x + \frac{1}{10}\, x^3$

d. $x - \frac{1}{10}\, x^3$

e. $x - x^3$

17. The inverse of the matrix $\begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix}$ is

a. $\begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}$

b. $\begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$

c. $\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{pmatrix}$

d. $\begin{pmatrix} -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$

e. $\begin{pmatrix} -\frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

18. Consider the system $\begin{cases} x_1 + 2x_2 + x_3 = 1 \\ -x_1 + x_2 = 0 \\ -2x_1 + 5x_2 + x_3 = 0 \end{cases}$

Which is true?

a. $\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 1 \end{cases}$ is a solution

b. It has infinitely many solutions.

c. The solution is given by $\begin{cases} x_1 = t \\ x_2 = t \\ x_3 = 1 - 3t \end{cases}$

d. It has no solutions.

e. None of the above

19. The eignevalues of the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ are given by

a. 1, 1, 3

b. 1, 3, 3

c. 1, 2, 3

d. 1, –3, 3

e. 1, –2, –3

20. (continued) For the largest eigenvalue, the corresponding eigenvector is

a. $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ b. $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ c. $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

d. $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ e. $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$