Math 226: Calculus IV Sign your Name:_____

Exam I *February 9, 1996* Print your Name:_____

Section:_____

TA:_____

Do not turn this page until you are told to begin.

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 16 multiple choice questions worth 6 points each. You start with 10 points, and the highest possible score is 106. Fill in the answers as you go along. You will not be allowed to fill in the answers after the time is up.

You may use a calculator, but only the standard functions found on very inexpensive scientific calculators. In particular you may not use graphing, integration, formula or program capabilities. Doing so is a violation of the Honor Code.

You are required to hand in the answer sheet and the problems. You may tear off the blank sheets at the back for scratch work. **Please tear off these scratch sheets gently, one page at a time.**

Which of the following differential equations is linear? (A) $y'' + siny = x$ **(B)** $y'' + yy' = y - x$ **(C)** $y' - \frac{1}{\sqrt{2x}}\ y = y - x$ **(D)** $x\ y' - 3y^2 = 0$ **Only C. Only A. Only B. Only D. None of them Solve the initial value problem**

$$xy' - 3y = x^5 \quad , \quad y(1) = 3/2.$$

Then find $y(2)$. 24 $\frac{39}{2}$ 20 16 32 **Find the largest in which the solution to the initial value problem**

$$x(x^2 - 9)y\prime + e^x y = \sqrt{4 - x^2}\ ,\ y(1) = 3$$

is certain to exist. $(0, 2)$ $(0, 3)$ $(-2, 2)$ **(-3, 2) (0,** ∞**) Classify the differential equation**

$$y\prime = \frac{sin(x) + y\ arctan(x)}{x^2} \quad x > 0.$$

Linear. Exact. Separable. Homogeneous. None of the others. Classify the differential equation

$$(xy + x)\ dx + (xy + y)\ dy = 0$$

Separable. Separable but not exact. Linear. Homogeneous. None of the others. Consider the equation

$$(x^2 - 1)y\prime = x^2)^{3/2}$$

For how many of the following points $(x_0\ ,\ y_0)$ **does the Existence-uniqueness Theorem guarantee a unique solution with the initial condistion** $(\mathbf{x}_0\ ,\ y_0) = y_0$ **?** (a) **(0,0)** (b) **(1,2)** (c) **(-1, -2)** (d) **(2,2)**

$One. Two Three Four None Suppose a radioactive substance decays at a rater proportional to the amount present. Find the time T un$

1/5 $of the original amount is left.$ **ln 5**$\frac{}{r}$ $\frac{lnr}{5}$ $\frac{ln5}{lnr}$ $\frac{lnr}{ln5}$ **A tank contains 100 pounds of salt dissolved in 200 gallons of water. Fresh water, i.e., water with no salt in it, enters the tank at a rate of 4 gallons per minute, with the well-stired misture leaving the tank at the same rate. After how many minutes will the tank contain 50 pounds of salt?** 34.66 138.63 25.02 7.82 29.44 **You invest** $\$2,000$ **in an account paying** $\$6\%$ **a year, compounded continously on the amount in excess of** $\$500$. **After ten years how much is in the account?** $\$3233.18$ $\$3186.27$

$3072.94**$2927.52$2989.48***The population equation* $\mathbf{dn}\frac{}{dt} = 4N - N^2$ **,** $N(0) = 2$, **has solution** $N(t) = \frac{4}{1+e^{-4t}}$ $\frac{4}{1+e^{-2t}}$ $\frac{2}{1+e^{-4t}}$ $\frac{e^{2t}}{4+e^{2t}}$ **Consider the equation**

$$N(t) = 4N(t) - N(t)^2 - 3\ ,\ t > 0,\ N(1) = 2$$

$N(t)$ **is concave up and** $\lim_{t\to\infty} N(t) = 3$ $N(t)$ **is concave down and** $\lim_{t\to\infty} N(t) = 0$ $N(t)$ **is concave up and** $\lim_{t\to\infty} \$N(t) = \infty$ $N(t)$ **is concave down and** $\lim_{t\to\infty} N(t) = -\infty$ **A 32 pound weight is dropping through a gas near the earth's surface. The resistance from the gas is equal to twice the square of the velocity. Find the magnitude of the terminal velocity of the body.** 2 4 6 8 10 **Consider the equation**

$$(s + y^2)\ dx + (2xy + 4y)dy = 0$$

find $f(x, y)$ **so that the equation** $f(x, y) = c$ **define solutions implicitly. Then compute** $f(1, 1) - f(0, 0)$. 4 0 -5 11 -1 **Which of the following is an integrating factor for the equation?**

$$(2xy - e^{2x})\ dx + xdy = 0$$

$\frac{e^2x}{x}$ $\mathbf{2xe}^x\mathbf{ye}^{-y}\mathbf{e}^2e^{-1/x}$

$xye^{2xy}Whichofthefollowingequationsdefinesimplicitlyorexplictlyasolutionoftheinitalvalueproblemy\prime = \frac{x^2+3y^2}{3xy}$, $y(1) = 3?\mathbf{y = x}\sqrt{9 + lnx}$ $y^2 = x^4+8$ $y = 3e^{(x-1)^2}\sqrt{x^2 + 3y^2} = lnx+\sqrt[2]{7}$ $e^{y-3} = 2x^2-1$ **Which of the following equations are NOT homogeneous? (A)** $(x^3y+x^2y^2)\ dy+xy^3dx = 0$ **(B)** $y\prime = e^{x-y}+x+y$ **(C)** $2xy\ y\prime = 4x^3 + 3y^3$ **(B) and (D) (A) and (C) (A) and (B) all three None**