Math 226: Calculus IV Sign your Name:____________________

Exam II *March 22, 1996* Print your Name:____________________

Section:____________________

TA:____________________

Do not turn this page until you are told to begin.

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 16 multiple choice questions worth 6 points each. You start with 10 points, and the highest possible score is 106. Fill in the answers as you go along. You will not be allowed to fill in the answers after the time is up.

You may use a calculator, but only the standard functions found on very inexpensive scientific calculators. In particular you may not use graphing, integration, formula or program capabilities. Doing so is a violation of the Honor Code.

You are required to hand in the answer sheet and the problems. You may tear off the blank sheets at the back for scratch work. **Please tear off these scratch sheets gently, one page at a time.**

Determine the largest interval in which the given initial value problem is certain to have a unique twice differentiable solution.

$(0, 2)$
$(-\infty, \infty)$
$(0, 4)$
$(2, \infty)$
$(0, \infty)$

Find the general solution of the differential equation

$$y'' + (y')^2 = 0$$

$y = ln|x + c_1| + c_2$
$y = \frac{1}{2}x^2 + c_1x + c_2$
$y = c_1 + c_2e^{-x}$
$y = c_1cos2x + c_2sin2x$
$y = ln|e_1x + c_2|$

Which of the following pair of functions are linearly <u>dependent</u> on the interval $0 < x < \infty$?

$e^x ande^{x-3}$
$sin\ x\ and\ cos\ x$
$ln\ x\ and\ ln\ (x + 2)$
$e^x and\ e^{5x}$
$x\ and\ x^2$

Find the general solutions of the equation

$$y'' - 2y' + 10y = 20$$

$y = e^x(c_1\ cos\ 3x + c_2\ sin\ 3x) + 2$
$y = e^{-x}(c_1\ cos\ 3x\ + c_2\ sin\ 3x) - 4$
$y = e^{3x}(c_1\ cos\ x + c_2\ sin\ x) + 2$
$y = c_1e^x + c_2e^{-3x} - 4$
$y = e^x\ cos\ 3x + c^x sin\ 3x$

Find $y(-1)$ if $y(x)$ is the solution to the initial value problem

$$y'' - 4y' + 4y = 0, y(0) = 1, y'(0) = 3$$

0 $2e^{-2}$ 3 e^{-2} $2e^2$

The function $y_1(x) = x^2 - 1$ is a solution to the equation

$$y'' + y' - \frac{2}{x-1}\ y = 0$$

The function $y = y_1|x|v|x|$ will be a record solution if $v|x|$ satisfies the differential equation $(x^2-1)v^{P''} + (x^2+4x-1)\ v^1 - 0$ $v'' + v' + v = 0$ $v'' + (x-2)\ v' = 0$ $v'' + (x^2-4x)\ v' - 0$ There is no such $v(x)$.

Find the general solution to the equation

$$y'' - y' = -2x$$

$y = c_1 + c_2\ e^x + 2x + x^2$ $y = c_1 e^x + c_2 x\ e^x + x^2 + 1$ $y = c_1 x + c_2\ e^x + x^2$ $y = c_1\ e^x + c_2 + 3x$ $y = c_1 + e_2\ xe^x + 2x$

The equation $y'' + \frac{1}{x}\ y' - \frac{1}{x^2} y = 0$ has solutions $y_1 = x$ and $y_2 = \frac{1}{x}$ for $x > 0$. Use the method of variation of parameters to find a solution of

$$y'' + \frac{1}{x} y' - \frac{1}{x^2} y = \frac{4}{x^3}$$

$y = 2\ \frac{2lnx}{x}$ $y = -4/x^2$ $y = 2\ lnx$ $y^2 = x^3$ $y = \frac{1}{x}$

A mechanical vibration is described by the differential equation

$$m\ \ u'' + \gamma\ u^1 + k\ u = 0$$

where $m > 0$, $\gamma\ \geq\ 0$, and $k > 0$. Choose the one true statement. If $\gamma^2 - 4\ km < 0$, then all solutions $u|t|$ have the value 0 for infinitely many t If $\gamma^2 - 4km = 0$, then are solutions $u(t)$ are periodic If $\gamma^2 - 4km > 0$, then all solutions $u(t)$ have the value 0 for infinitley many t For a certain choice of γ, all solutions $u(t)$ become unbounded as $t \longrightarrow\ \infty$. For a certain choice of $\sqrt{\frac{k}{m}}$, all solutions $u(t)$ become unbounded as $t \longrightarrow \infty$.

cos $wot + \sqrt{3} sin\ wot$ is equal to $2cos\ (wot - \pi/3)$ $2\cos(wo - \frac{\pi}{3})t$ $\sqrt{3}\ cos(wot - \pi/6)$ $2\ \cos(wot + \pi/6)$ $\sqrt{3}\ cos(wot - \pi/3)$

The coefficients of a power series solution $y|x| = \Sigma_{m=0}^{\infty}$ an ? satisfy the recurrence relation $an{+}2 = -\frac{1}{n=2}$ an for ≥ 0. Find the solution which fulfills the initial condition $y(0) = 0, y'(0) = 1$ $\frac{x-1}{3}$ $x^3 + \frac{1}{5.3}x^5 - \frac{1}{7.5.3}x^7 + ...$ $-\frac{1}{3}x^3 + \frac{1}{5.3}x^5 - \frac{1}{7.5.3}x^7 + \frac{1}{9.7.5.3}x^9 + ...$ $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$ $x - \frac{1}{3}x^2 + \frac{1}{5.3}x^3 - \frac{1}{7.5.3}x^4 + ...$ $1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + ...$

Using the method of undeterminied coefficients, find the correct from for a particular solution y of the differential equation.

$$y'' + y' = e^2 x + 5\ sin\ 2x$$

$y = Ae^{2x} + x(B\ cos\ 2x\ + C\ sin\ 2x)$ $y = Axe^{2x} + Bx\ sin\ 2x$ $y = Ae^{2x}\ + B\ cos\ 2x + C\ sin\ 2x$ $y = Ae^{2x} + (Bx + C) sin\ 2x$ $Y = xe^{2x}(A\ cos\ 2x + B\ sin\ 2x)$

The equation $(x^2 - 4x + 5)\ y'' + x^3 y' - (4x-3)y = 0$ has a power series solution in powers of $(x-4)$. From the general theory its radius of convergence is at least

$\sqrt{5}$ $-2+i$ ∞ 4 5

The equation $y'' - 3xy' - 3y = 0$ has two linearly independent solutions expressed in powers of x. Find the recurrence relation that determines the coefficients $an = 2 = \frac{3an}{n+2}\ for\ n \geq$ $an + 1 = \frac{3an}{n+1}\ for\ n \geq 0$ $an + 2 = -\frac{an}{(n=1)(n)}\ for\ n \geq 1\ and\ a_2 + a_0.$ $an + 2 = \frac{3an{+}1{+}an}{n=2}\ for\ n \geq 0$ $an + 1 = \frac{2an}{n=2}\ \ for\ \ n \geq 0$

Consider the singular points $x = 0\ and\ x = 1$ for the equation

$$x^3(x-1)^2\ y'' + (x^3 - x^2)\ y'\ +\ (x-2)\ y = 0$$

0 is irregular and 1 is regular. Both are regular. Both are irregular. 0 is regular and 1 is irregular. Both are ordinary points.

Find the general solution of the Euler equation

$$x^2 y''\ - 5xy'\ +\ 9y = 0,\ x > 0$$

$x^3(c_1 + c_2\ ln\ x)$ $x^{5/2}\ (c_1\ cos\ \frac{3}{2}\ ln\ x)\ +\ c_2\ sin\ (\frac{3}{2}\ ln\ x)$ $c_1\ x^3\ +\ c_2\ x^4$ $c_1\ cos(\frac{3}{2}\ ln\ x)\ +\ c_2\ sin\ (\frac{3}{2}\ ln\ x)$ $c_1\ x^{5/2}\ +\ c_2\ x^{3/2}$