Math 226: Calculus IV Sign your Name:________________

Exam III *April 19, 1996* Print your Name:________________

Section:________________

TA:________________

Do not turn this page until you are told to begin.

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 16 multiple choice questions worth 6 points each. You start with 10 points, and the highest possible score is 106. Fill in the answers as you go along. You will not be allowed to fill in the answers after the time is up.

You may use a calculator, but only the standard functions found on very inexpensive scientific calculators. In particular you may not use graphing, integration, formula or program capabilities. Doing so is a violation of the Honor Code.

You are required to hand in the answer sheet and the problems. You may tear off the blank sheets at the back for scratch work. **Please tear off these scratch sheets gently, one page at a time.**

Find the exponents of the singularity .

$2\ and -1\sqrt{2}$ and -$\sqrt{2}$ $2and1$ $0and0$ $3and-1$

The equation $x^2y\prime\prime\ +\ (x^3+x)y\prime - 4y = 0$ has a regular singular point at $x = 0$. For which values of γ is there sure to be a series solution of the form

$$y(x) = |x|^\gamma \sum_{n=0}^{\infty} a_n x^n\ ,\ \text{with } a\ 0 \neq 0?$$

2 $2and-2$ $-\frac{1}{2}\ \pm\ \sqrt{\frac{17}{2}}$ $1and-4$ 1

The equation $x^2y\prime\prime - x(x+3)y\prime + (x+3)\ y = 0$ has a regular singular point at $x = 0$ and the exponents of the singularity are 3 and 1. The recurrence relation is

$$a_n = \frac{(n+\gamma-2)a_n - 1}{(n+\gamma-1)(n+\gamma-3)}\ ,\quad for\ m \geq 1.$$

Find the first three nonzero terms of a series solution near $x = 0$, with $a_0 = 1$.

$x^3(1+\frac{2x}{3}\ +\ \frac{x^2}{4}\ +\ldots)$ $\frac{2x4}{3}\ +\ \frac{x^5}{4}\ +\ \frac{s^6}{15}\ +\ldots$ $1+\frac{2x}{3}+\ \frac{3x^2}{4}\ +\ldots.$ $1+\frac{x}{3}\ +\ \frac{x^3}{15}\ +\ldots$ $x+\frac{2x^2}{3}\ +\ \frac{x^3}{4}\ +\ldots$

The equation $2x^2\ y\prime\prime\ +\ 3xy\prime\ -\ (1+x)\ y = 0$ has a regular singular point at $x = 0$, with indicial equation $(\gamma+1)(2\gamma-1) = 0$. Find the recurrence relation for $n \geq 1$ for the coefficients of the series solution corresponding to $\gamma = \frac{1}{2}$

$a_n =\ \frac{a_n-1}{2n^2+3n}$ $a_n =\ \frac{a_n-2}{n^2+3^n}$ $a_n =\ \frac{a_n-1}{2n+1}$ $a_n =\ \frac{a_n-1}{n(n+3)}$ $a_n =\ \frac{a_n-2}{xn(n+1)}$

For what values of c and d is the following system consistent?

$$\begin{array}{ccccccccc} x_1 & + & x_2 & + & x_3 & & & = 2 \\ & & x_2 & & & + & x_4 & = 1 \\ 2x_1 & + & 3x_2 & + & 2x_3 & + & x_4 & = c \\ x_1 & & & + & x_3 & - & x_4 & = d \end{array}$$

$c = 5, d = 1$. Consistent for every c and d. $c = 4, d = 2$ $c = 0, d = 3$ $c = \alpha,\ d = 1-\alpha$ for α in $\Re$.

Let W be the subspace of $\Re^5$ spanned by the vectors (1, 0, 1, 0, 1), (1, 1, 1, 1, 1), (1,2, 1, 2, 1) and (2,3, 2, 3, 2) Find the dimension of W.

2 1 3 4 5

Let A be a 7x11 matrix. Suppose upon reduction to row echelen form, one obtains exactly 5 nonzero rows. What is the dimension of the column space of A?

5 6 2 1 4

Let W be the set of solutions of A x = 0

$$A = \begin{bmatrix} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Which of the folowing is a basis for W?

$(-1,3,1,0,0) and (2,0,0,1,0)$ $(1,0,1,-2,0) and (0,1,-3,0,2) and (0,0,0,0,1)$ $(s,3,1,1,0)$ $(1,-3,0) and (0,2,1)$ $(1,-3,1,2,0) and (2,0,0,0,1)$

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

It's first row is

$(0,-1,0)$. $(2,0,1)$. It has no inverse. $(0,1,0)$. $(-1,-1,1)$.

For $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, find the last row of A B.

$(5,-9)$ The product is not defined $(4,6)$ $(0,8)$ $(3,-12)$

Let A x = 0 be an m x n system of honogeneous equations of rank γ. Consider the following statements

A. There is always at least one solution.

B. If $\gamma < n$, there are infinitely many solutions

C. If $\gamma < m$, there are infinitely many soutions.

D. If $\gamma = n$, there is exactly one solution.

E. If $\gamma = m$, there is exactly one solution.

Only A, B, C and D are true Only A is true Only A and B are true Only A, B and C are true All are true

Let A = $\begin{bmatrix} 2 & 0 & 3 \\ 1 & -4 & 5 \end{bmatrix}$ Find the middle row of $A^t A$.

$(-4,16,-20)$ $A^t A$ is not defined. $(5,-4,7)$ $(10,-20,34)$ $(11,16,8)$

If $A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, find the bottom row of $(AB)^{-1}$

$(1,1,2)$ $(0,3,2)$ A B is not invertible $(2,0,3)$ $(1,2,1)$

Transform the following matrix into reduced echelon form.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of the subsets

$W_1 = \{(x_1, x_2, 0) : x_1^2 + x_2^2 = 1\}$

$W_2 = \{(x_1, x_2, x_3) : 3x_1 + 4x_2 - 5x_3 = 0\}$

$W_3 = \{(x_1, x_2, x_3) : x_1 = 0\gamma \; x_2 = 0\}$

are subspaces of $\Re^3$?

Only W_2 Only W_1 Only W_3 Only W_1 *and* W_2 Only W_2 *and* W_3
Considered as vectors in $\mathcal{M}$ 2x2 ,

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , \begin{pmatrix} 1 & 1 \\ 1 & k \end{pmatrix}$$

are Linearly dependent for any choice of k. Linearly independent for any choice of k. Linearly dependent only if k = 1. Linearly independent only if k = 1. Linearly dependeant only for k = 0.