Name: _____ **Instructor:** Heide Gluesing-Luerssen

Mathematics 226.02: Differential Equations and Linear Algebra
Fall Semester 1997
Exam 1
September 22, 1997

This Examination contains 5 problems, each worth 20 points, on 7 sheets of paper including the front cover and one extra sheet on the back. Do all your work in this booklet and show your computations. Calculators, books, and notes are not allowed.

Scores

Question	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Sign the pledge:
"On my honor, I have neither given nor received unauthorized aid on this Exam."

Signature: _____

GOOD LUCK

1. Solve the following initial value problem and determine the interval, where the solution is defined.

$$(x^2 - 4)\frac{dy}{dx} + 2xy = \frac{2x^2 - 8}{x}, \qquad y(1) = 2$$

Answer: _____

2. Find the value of a, for which the equation is exact, and then solve it using that value of a.

$$(y^2 + x^3 + 2x) + (axy + y^2 + 3)\frac{dy}{dx} = 0$$

Answer: _____

3. (a) Solve the differential equation

$$x^3\frac{dy}{dx} + (y-2)^2 = 0$$

Answer: _____

(b) Find an implicit expression for the solution of the differential equation

$$x\frac{dy}{dx} = y + 2xe^{\frac{y}{x}}$$

Answer: _____

4. (a) A skydiver weighing 160 lbs. (including equipment) falls vertically down from a certain high altitude. The parachute opens when the skydiver reaches speed equal to 130 ft/sec. Assume that $t = 0$ when the parachute opens and that the air resistance is $10|v|$ when the parachute is open. Assume also that the gravitational force is 32 ft/sec^2.

(i) Write the initial value problem (i. e. a differential equation and an initial condition) for the speed $v(t)$ at any time $t > 0$. **DO NOT SOLVE IT!**

(ii) Find the limiting velocity v_L of the skydiver after the parachute opens.

Answer: _____

(b) Circle the differential equation whose direction field is shown in the following picture.

A. $y' = y(x + y)$ B. $y' = (x - y)(x + y)$ C. $y' = (2 - y)(x + y)$

D. $y' = (y - 2)(x + y)$ E. $(x - 2)(x + y)$

5. (a) Consider a tank holding initially 500 gallons of a salt solution with concentration 0.3 lb of salt per gallon. A solution containing 0.4 lb of salt per gallon is pumped into the tank at a rate of 3 gallons per minute, and the well-stirred mixture flows out of the tank at a rate of 5 gallons per minute. Write the initial value problem (i. e. a differential equation and an initial condition) needed to find the amount $S(t)$ of salt in the tank at time $t > 0$ prior to the instant when the tank is empty. **DO NOT SOLVE IT!**

Answer: _____

(b) Find the constant (equilibrium) solutions of the differential equation

$$\frac{dy}{dt} = -0.002(1 - \frac{y}{3})(1 - \frac{y}{7})y$$

and classify each one as asymptotically stable or unstable. Sketch the graphs of the four solutions y_1, y_2, y_3, y_4, which have initial values $y_1(0) = 3$, $y_2(0) = 4$, $y_3(0) = 7$, $y_4(0) = 1$.