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This Examination contains five problems worth a total of 100 points, each problem worth 20 points, on (7) sheets of paper including the front cover and one extra sheet on the back. Do all your work in this booklet and show your computations. Calculators, books and notes are not allowed.

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Total	

**Sign the pledge:** “On my honor, I have neither given nor received unauthorized aid on this Test”:

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**GOOD LUCK**

1. Find the general solution of the differential equations:

$$xdydx + (y + 1)^2 = 01.1$$

**Answer:** \_\_\_\_\_

$$xdydx = y - xe^{y/x} 1.2$$

**Answer:** \_\_\_\_\_

**2.** Solve the following initial value problem:

$$(1 + x^4)dydx + 4x^3y = 1, \quad y(0) = 7.$$

**Answer:** \_\_\_\_\_

- 3.** Find  $a$  for which the following differential equation is exact and then solve it.

$$(axy + x^2 + 3) + (x^2 + y^3 + 2y)dydx = 0$$

**Answer:** \_\_\_\_\_

- 4a.** A skydiver weighing 192 lbs.( including equipment) falls vertically downward from a certain high altitude. The parachute opens when the skydiver reaches speed equal to 160 ft/sec. Assume that  $t = 0$  when the parachute opens and that the air resistance is  $12|v|$ . Also assume that  $g = 32 \text{ ft/sec}^2$ .

- i) Write the initial value problem for the speed  $v(t)$  at any time  $t$ . **DO NOT SOLVE IT!**  
ii) Find the limiting velocity  $v_L$  of the skydiver after the parachute opens.

**Answer:** \_\_\_\_\_

- 4b.** Find the constant (equilibrium) solutions of the differential equation

$$dy/dt = -0.01(1 - y^2)(1 - y^8)y.$$

Then classify each one as asymptotically stable, or unstable, and sketch their graphs. Finally sketch the graph of the solution  $y = y(t)$  with initial value  $y(0) = 3$  (indicate its behavior for large  $t$ ).

- 5a.** Consider a tank holding initially 100 gallons of a salt solution with concentration 0.2 lb of salt per gallon. A solution containing 0.4 lb of salt per gallon is pumped into the tank at the rate of 5 gal/min, and the well-stirred mixture flows out of the tank at the rate of 2 gal/min. Write the initial value problem (i.e. a differential equation and an initial

condition) needed to find the amount  $S(t)$  of salt in the tank at any time  $t$ . **DO NOT SOLVE.**

**Answer:** \_\_\_\_\_

**5b.** Circle the differential equation whose direction fields are shown in the following picture.

A.  $y' = y(y-x)$ ,   B.  $y' = (y-1)x$ ,   C.  $y' = (y-1)(x-y)$ ,   D.  $y' = 1-x^2$ ,   E.  $y' = \sin x$

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