Name: _____ **Instructor:** Heide Gluesing-Luerssen

Math 226: Differential Equations and Linear Algebra
Fall Semester 1998
Exam 1
September 28, 1998

This Examination contains 8 problems on 7 sheets of paper including the front cover. Do all your work in this booklet and show your computations. Calculators, books, and notes are not allowed.

Scores

Question	Possible	Actual
1	15	
2	15	
3	15	
4	5	
5	15	
6	15	
7	10	
8	10	
Total	100	

Sign the pledge:
"On my honor, I have neither given nor received unauthorized aid on this Exam."

Signature: _____

GOOD LUCK

1. Solve the following initial value problem and determine the interval where the solution is defined.

$$(t^2 - 9)\frac{dy}{dt} + 2ty = \frac{2t^2 - 18}{t^2}, \quad y(1) = -1.$$

Answer: _____

2. Find the value of a, for which the equation is exact, and then solve it using that value of a.

$$(3x^2 - axy^2 + 2y)dx + (3y^2 - 2x^2y + 2x)dy = 0.$$

You may leave your answer in implicit form.

Answer: _____

3. Find the general solution of the equation

$$2x\frac{dy}{dx} = 2y + xe^{\frac{y}{x}}.$$

You may leave your answer in implicit form.

Answer: _____

4. (5 points, no partial credit) Circle the differential equation whose direction field is shown in the following picture.

A. $y' = y(x - y)$ **B.** $y' = (x - y)(x + y)$ **C.** $y' = (1 - y)(x - y)$

D. $y' = (y - 1)(x - y)$ **E.** $y' = (x - 1)(x - y)$

5. A tank initially contains 300 gallons of pure water. A mixture containing 1 lb of salt per gallon enters the tank at a rate of 3 gal/min. The well-stirred mixture leaves the tank at a rate of 5 gal/min.
Write the initial value problem needed to find the amount of salt $S(t)$ in the tank at time $t > 0$ prior to the instant when the tank is empty. **Do not solve it!**
[Hint: First you should figure out how many gallons of mixture are in the tank at time t.]

Answer: _____

6. Given the differential equation

$$\frac{dy}{dt} = 2y(y-1)(y-3).$$

Do not attempt to solve it!

(a) Find the (constant) equilibrium solutions and classify each one as asymptotically stable, unstable, or semistable.

(b) Sketch the graphs of the four solutions $y_1,\ y_2,\ y_3$, and y_4 which have initial condition $y_1(0) = 1,\ y_2(0) = 2,\ y_3(0) = 3$, and $y_4(0) = 0.5$. Neglect concavity.

7. Find the solution of the initial value problem

$$6y'' - 7y' + y = 0, \quad y(0) = 0, \quad y'(0) = 5.$$

Answer: _____

8. Determine the largest interval in which the following initial value problem is certain to have a unique twice differentiable solution.

$$(t-3)(t-1)ty'' + t^2y' + (t-1)y = \sin t, \quad y(2) = -3, \quad y'(2) = 4.$$

Do not attempt to find the solution!

Answer: _____