Name: ________________________________ **Instructor:** Heide Gluesing-Luerssen

Math 226: Differential Equations and Linear Algebra
Fall Semester 1998
Exam 2
November 9, 1998

This Examination contains 7 problems on 8 sheets of paper including the front cover (not counting 2 blank pages at the end). Do all your work in this booklet and show your computations. Calculators, books, and notes are not allowed.

Scores

Question	Possible	Actual
1	15	
2	15	
3	15	
4	15	
5	15	
6	10	
7	15	
Total	100	

Sign the pledge:
"On my honor, I have neither given nor received unauthorized aid on this Exam."

Signature: ________________________________

GOOD LUCK

1. Solve the following initial value problem.

$$y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

Your answer should be in real form (i.e. no complex numbers). (15 points)

Answer: ____

2. Find the general solution $y(x)$ of

$$y'' - 4y' + 4y = 3e^{-3x}.$$

(15 points)

Answer: ______________________________

3. Consider the following three pairs of functions.

A. $3\sin x,\ 4\cos x$;

B. $\sin x,\ 2\sin x$;

C. $x^2,\ 3x^4$.

(a) Compute the Wronskian of each pair. (5 points)

(b) Which of the pairs are linearly independent? Explain briefly. (5 points)

(c) Which pairs consist of functions that could not both solve the same differential equation

$$y'' + p(x)y' + q(x)y = 0, \quad -\infty < x < \infty$$

(assume p and q are continuous everywhere)? Explain briefly. (5 points)

4. The function $y(x) = 1/x$ solves the differential equation

$$xy'' + (2 - 2x)y' - 2y = 0.$$

Find the general solution to the equation. (15 points)

Answer: ______________________________

5. Consider the solution $y(x)$ of the initial value problem

$$y'' + 2xy = 0, \quad y(0) = 2, \quad y'(0) = 3.$$

Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series representing y.

(a) Write down a recurrence relation for finding the coefficients a_n. (10 points)

Answer: ________________________________

(b) Compute the values of the coefficients a_0, a_1, a_2, a_3. (5 points)

Answer: ________________________________

6. The equation

$$(x^2 - 3x)y'' + xy' + (x - 2)y = 0$$

has a power series solution $y = \sum_{n=0}^{\infty} a_n(x-2)^n$. What is the largest interval on which this series is certain to converge? Make sure your work explains your answer. (10 points)

Answer: ______________________________

7. Suppose that you are designing the suspension system for a new car. The 1000 kilogram vehicle is mounted on its springs without any shock absorbers (i.e. without damping). As a test, you push the car down and let it bounce back up. It bobs up and down about once a second.

(a) Write down a differential equation that describes the motion of this undamped system. Compute the values of the coefficients occurring in the equation. (10 points)

Answer: ______________________________________

(b) What sort of damping would the shock absorbers need to provide (i.e. what would the damping constant need to be) in order to make this system critically damped? (5 points)

Answer: ______________________________________