1. Solve the equation $2(y-4x^2)dx + xdy = 0$.

(a) $y = 2x^2 + \frac{C}{x^2}$ (b) $y = \frac{2}{x^2} + Cx^2$ (c) $y = 2x^4 + Cx^2$ (d) $y = \frac{8x^3}{3} - \frac{x^2}{2} + C$ (e) $y = 8x^2(\ln|x| + C)$

2. Find the solution of the initial value problem $ty' + (1-t)y = e^t, \quad y(1) = 0.$

(a) $e^t(1 - \frac{1}{t})$ (b) $e^{-t}(1-t)$ (c) $e^{t-1} - \frac{1}{t}$ (d) $\frac{1}{t} - e^{1-t}$ (e) $e^{-t}(\frac{1}{t} - 1)$

3. Find the largest interval in which a unique solution to the initial value problem

$$(t^2 + 2t)y' + (t^2 - 2t)y = e^{-(t-3)}, \quad y(1) = 3,$$

is certain to exist.

(a) $(0, \infty)$ (b) $(0, 2)$ (c) $(-2, 2)$ (d) $(-2, 3)$ (e) $(-\infty, 2)$

4. Solve the initial value problem

$$y' + 2y^2 = 0, \quad y(0) = \frac{1}{2},$$

and then give the domain of the solution.

(a) $(-1, \infty)$ (b) $(-\infty, 1)$ (c) $(-2, \infty)$ (d) $(-\infty, 2)$ (e) $(-1, 1)$

5. Consider the equation $\frac{dy}{dt} = \frac{y+t}{t(y-t)^2}$. For which of the following points (t_0, y_0) does the Existence-Uniqueness Theorem guarantee a unique solution with the initial condition $y(t_0) = y_0$?

$$(1, 0) \qquad (0, 1) \qquad (1, 1) \qquad (-1, 1)$$

(a) Only $(1, 0)$ and $(-1, 1)$. (b) Only $(1, 0)$ and $(0, 1)$. (c) Only $(0, 1)$ and $(1, 1)$. (d) Only $(1, 0)$ and $(1, 1)$. (e) Only $(0, 1)$ and $(-1, 1)$.

6. A certain radioactive material has a half-life of 200 years. What percent of a given sample will remain after fifty years?

(a) 84% (b) 79% (c) 87% (d) 92% (e) 90%

7. The sum of \$10,000 is borrowed at the annual interest rate of 10.8%, which we assume to be compounded continuously. If the loan is paid off at the rate of \$225 per month, how many years will it take to pay off the loan?

   (a) 4.73 (b) 4.55 (c) 4.13 (d) 3.98 (e) 4.27

8. A 200 gallon tank initially has a 30% concentration of salt.

   Salt solution enters the tank from two different sources.

   One source provides water containing one pound of salt per gallon at a rate of two gallons per minute.

   A second source provides three gallons per minute at a varying concentration of $2e^{-2t}$ pounds of salt per gallon.

   The contents are stirred and the uniform solution is drained off at a rate of five gallons per minute.

   Which initial value problem is satisfied by the quantity $Q(t)$ of salt in the tank at time t?

   (a) $Q' + \frac{1}{40}Q = 2 + 6e^{-2t}, \quad Q(0) = 60.$ (b) $Q' + \frac{1}{40}Q = 5 + 10e^{-2t}, \quad Q(0) = 60.$ (c) $Q' = -55 + 10e^{-2t}, \quad Q(0) = 30.$ (d) $Q' = \frac{79}{40} + 6e^{-2t}, \quad Q(0) = 0.30.$ (e) $Q' + \frac{1}{40}Q = 5e^{-2t}, \quad Q(0) = 60.$

9. An object weighing 200 pounds falls in a medium offering a resistive force equal to $-kv^{1.2}$, where v is the velocity. The terminal velocity is measured to be 17.5 feet per second. Estimate k.

   (a) 6.45 (b) 2.11 (c) 10.42 (d) 5.28 (e) 13.64

10. Classify the equilibrium solutions of the equation $\frac{dy}{dt} = y(1-y)^2$ as stable, semistable or unstable.

    (a) $y = 0$ is unstable and $y = 1$ is semistable. (b) Both $y = 0$ and $y = 1$ are semistable. (c) $y = 0$ is unstable and $y = 1$ is stable. (d) $y = 0$ is semistable and $y = 1$ is unstable. (e) $y = 0$ is stable and $y = 1$ is unstable.

11. Solve the population equation $y' = 3(y-1)(y-2), \quad y(0) = 0.$

    (a) $y = \frac{2-2e^{3t}}{1-2e^{3t}}$ (b) $y = \frac{1-e^{3t}}{1+e^{3t}}$ (c) $y = \frac{2-2e^{3t}}{2+e^{3t}}$ (d) $y = \frac{e^{3t}-1}{2e^{3t}+1}$ (e) $y = \frac{2-2e^{-3t}}{1-2e^{-3t}}$

12. If $(xy^2 + y - x)dx + (x^2y + x + 2)dy = 0$, and $y = 1$ when $x = 2$, find y when $x = 0$.

(a) 2 (b) -3 (c) $\frac{1}{2}$ (d) -1 (e) 1

13. Classify the following equation.

$$(\frac{y}{x^2 + y^2} - \frac{y^2}{x^2})dx + (2\frac{y}{x} - \frac{x}{x^2 + y^2})dy = 0$$

(a) Exact but not separable. (b) Separable. (c) Linear. (d) Homogeneous. (e) None of the others.

14. Which of the following is an integrating factor for the equation $\frac{dy}{dx} = -\frac{4xy+3y^2-x}{x^2+2xy}$

(a) x^2 (b) $2x$ (c) e^{2x} (d) $\frac{1}{x^3}$ (e) e^{-2x}

15. Classify the equation $\frac{dy}{dx} = (x + y)^2$.

(a) None of the others. (b) Linear. (c) Separable. (d) Exact but not separable. (e) Homogeneous.

16. Determine which of the following defines a solution to the the equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$.

(a) $y = x\tan(\ln|x| + C)$ (b) $x^2 + y^2 = Cx^3$ (c) $\frac{x}{x+y} = -\ln|x| + C$ (d) $\arcsin(1 + \frac{y}{x}) = C\ln|x|$
(e) $y = x + x\tan(x + C)$

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Do not turn this page until you are told to begin. **Do not detach the answer sheet from the test.** You are required to hand in both the answer sheet and the problems. Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 16 multiple choice questions worth 6 points each. You start with 10 points, and the highest possible score is 106. Fill in the answers as you go along. You will not be allowed to fill in the answers after the time is up. You may use a calculator, but only the standard functions found on very inexpensive scientific calculators. In particular you may not use graphing, integration, formula, matrix or program capabilities.

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70pt Answer Key 1

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