Math 226: Calculus IV Sign your Name:____**Exam II**

March 20, 1998 Print your Name:____

Section:____ TA:____

Do not turn this page until you are told to begin. **Do not detach the answer sheet from the test.** You are required to hand in both the answer sheet and the problems. Record your

answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 16 multiple choice questions worth 6 points each. You start with 10 points, and the highest possible score is 106. Fill in the answers as you go along. You will not be allowed to fill in the answers after the time is up. You may use a calculator, but only the standard

functions found on very inexpensive scientific calculators. In particular you may not use graphing, integration, formula, matrix or program capabilities.

Let $y(t)$ be the solution of $\quad 2y'' + y' = 0 \quad$ for which $\quad y(0) = y'(0) = 1$. Find $\lim_{t\to\infty} y(t)$. 3 $\frac{3}{2}$ 1 4 ∞

The existence and uniqueness theorem for second order linear differential equations guarantees a unique solution to the equation

$$(t-1)^2 y'' + (\ln t)y' + 3y = 2$$

satisfying $y(2) = 1, y'(2) = 0$ $y(0) = 1, y'(0) = 2$ $y(1) = 2, y'(1) = 3$ $y(3) = 2, y(5) = 4$ $y(0) = 0, y'(0) = 0$

Each of the following is a set of solutions of the equation $\quad y'' - 9y = 0$. Find the one that is **NOT** a fundamental set of solutions. $\{e^{3t-1}, e^{3t+1}\}$ $\{e^{3t}, e^{-3t}\}$ $\{e^{3t} + e^{-3t}, e^{3t} - e^{-3t}\}$ $\{e^{1-3t}, e^{1+3t}\}$ $\{3e^{3t}, -e^{-3t}\}$

Let $W(t)$ be the Wronskian of a fundamental set of solutions of the equation

$$(1 + t^2)y'' + ty' - (3t + 2)y = 0.$$

If $W(0) = 4$, find $W(\sqrt{3})$. 2 1 8 0.707 -4

Let $y(t)$ be the solution of the initial value problem

$$y'' - 2y' + 17y = 0, \quad y(0) = 1 = y'(0).$$

Find $y(\frac{\pi}{16})$. 0.861 1.721 -2.139 1.217 0

Find the general solution of the equation $\quad 4y'' + 12y' + 9y = 3e^{\frac{3}{2}t}$. $e^{-\frac{3}{2}t}(c_1 + c_2 t) + \frac{1}{12}e^{\frac{3}{2}t}$ $e^{-\frac{3}{2}t}(c_1 + c_2 t) + \frac{t}{6}e^{\frac{3}{2}t}$ $e^{-\frac{3}{2}t}(c_1 + c_2 t + \frac{t^2}{9})$ $c_1 e^{-\frac{3}{2}t} + c_2 t + \frac{1}{18}e^{\frac{3}{2}t}$ $e^{\frac{2}{3}t}(c_1 + c_2 t) - \frac{1}{12}e^{\frac{3}{2}t}$

The function $y_1(t) = e^t$ is a solution of the differential equation

$$ty'' - y' - (t-1)y = 0, \quad t > 0.$$

The function $y_1(t)v(t)$ will be a second linearly independent solution if $v(t)$ satisfies the differential equation $v'' + (2 - \frac{1}{t})v' = 0$. $tv'' + (1-2t)v' = 0$. $v'' + \frac{e^{2t}}{t}v' = 0$. $(te^{-2t}v')' = 0$. $v'' + (1 + \frac{1}{t})v' = 0$.

Using the method of undetermined coefficients, find the correct form for a particular solution $Y(t)$ of the equation

$$y'' + 4y = e^{2t}\cos 2t - \sin 2t.$$

$e^{2t}(A\cos 2t + B\sin 2t) + t(C\cos 2t + D\sin 2t)$ $e^{2t}(A\cos 2t + B\sin 2t) + C\cos 2t + D\sin 2t$ $te^{2t}(A\cos 2t + B\sin 2t)$ $(At+B)e^{2t}\cos 2t + (Ct+D)\sin 2t$ $te^{2t}(A\cos 2t + B\sin 2t) + t(C\cos 2t + D\sin 2t)$

Suppose that $y_1(x) = x$ and $y_2(x) = x^2$ are solutions of the differential equation

$$y'' + p(x)y' + q(x)y = 0,$$

where $p(x)$ and $q(x)$ are continuous for $x > 0$. Find a particular solution to the nonhomogeneous equation

$$y'' + p(x)y' + q(x)y = 10x^4.$$

$\frac{x^6}{2}$ $\frac{x^5}{10}$ $\frac{x^6}{5}$ $\frac{x^5}{12}$ $\frac{x^7}{20}$

An object which weighs 32 pounds is attached to a spring with spring constant 1 pound/foot and is immersed in a viscous medium with damping constant 2 pound-seconds/foot. At time $t = 0$ the mass is pulled down $\frac{1}{4}$ foot from the equilibrium position and given an initial velocity of one foot/second in the downward direction. Solve the spring equation and choose the best description of what happens. (g=32.) The mass will move away from and then slowly return toward the equilibrium position without ever passing through it. The mass will pass through the equilibrium position once, and then slowly return toward it. The mass will oscillate about the equilibrium position, moving the same distance above and below it over and over. The mass will pass through the equilibrium position exactly twice. The mass will oscillate about the equilibrium position, but with constantly decreasing distance above and below it.

The expression $-2\cos 5t + 7\sin 5t$ can be rewritten in the form $R\cos(5t - \delta)$, where an approximate value for δ in radians is 1.849 1.292 4.990 2.863 -1.292

For the equation $(1-x^2)y'' - 6xy' - 4y = 0$, find the first three nonzero terms of the series solution in powers of x for which $y(0) = 1$ and $y'(0) = 0$. $1 + 2x^2 + 3x^4$ $2 + \frac{5}{3}x^3 + \frac{7}{3}x^5$ $x^2 + 3x^4 + 4x^6$ $1 + x + 2x^2$ $1 + x^2 + \frac{3}{2}x^4$

Find the recurrence relation for the series solution in powers of $\mathbf{x-1}$ of the equation

$$(2x - x^2)y'' - y = 0.$$

$a_{n+2} = \frac{n^2-n+1}{n^2+3n+2}a_n$ $a_{n+2} = \frac{n-1}{n+2}a_n$ $a_{n+2} = \frac{(n-1)a_{n-1}+a_n}{(n+2)(n+1)}$ $a_{n+2} = \frac{n+1}{n+2}a_n$ $a_{n+2} = \frac{(n-1)a_{n-1}+na_n}{(n+2)(n+1)}$

The equation $\quad (x-2)(x^2+4x+8)y''+(x^2+2)y'+(x-2)y=0$
has a power series solution in powers of $x+2$. The general theory guarantees that its radius of convergence is at least 2. $2\sqrt{2}$. $2\sqrt{5}$. 4.

$3\sqrt{2}$.

The equation $\quad (x^2-1)^2y''+(x^2-1)y'+y=0 \quad$ has two regular singular points. two irregular singular points. one regular singular point and one irregular singular point. just one singular point, which is regular. just one singular point, which is irregular.

Find the general solution of the Euler equation

$$x^2y''+7xy'+13y=0.$$

$x^{-3}(c_1\cos(2\ln|x|)+c_2\sin(2\ln|x|))$ $x^3(c_1\cos(\ln|x|)+c_2\sin(\ln|x|))$ $x^{-3}(c_1\cos(\ln 2|x|)+c_2\sin(\ln 2|x|))$ $|x|^{-\frac{7}{2}}(c_1\cos(\frac{\sqrt{3}}{2}\ln|x|)+c_2\sin(\frac{\sqrt{3}}{2}\ln|x|))$ $|x|^{-\frac{7}{2}}(c_1\cos(\ln\frac{\sqrt{3}}{2}|x|)+c_2\sin(\ln\frac{\sqrt{3}}{2}|x|))$