Math 226: Calculus IV Sign your Name:_________________________________**Final Exam**

May 7, 1998 Print your Name:________________________________ Section:________________________________

TA:________________________________ Do not turn this page until you are told to begin. **When you are told to begin, tear off the answer sheet (gently) and keep it under your test while you are working. At the end of the exam please turn in only the answer sheet.**

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this answer sheet. There are 22 multiple choice questions worth 6 points each. You start with 24 points, and the highest possible score is 156. Fill in the answers as you go along. You will not be allowed to fill in the answers after the time is up. You may use a calculator, but only the standard functions found on very inexpensive scientific calculators. In particular you may not use graphing, integration, formula or program capabilities.

Let $y(t)$ be a solution of the equation $t^2y' - y = -1$ defined for $t > 0$. Find $\lim_{t\to 0^+} y(t)$. 1 0 ∞ -1 It can be different for different solutions.

Classify the stable equilibrium points for the equation

$$\frac{dN}{dt} = \begin{vmatrix} N^2 & 0 & 2N-2 \\ 0 & 1 & 0 \\ 2N-2 & 0 & (N-1)^2 \end{vmatrix}.$$

One stable, one unstable, and one semistable. Two stable, one unstable, and none semistable. One stable, two unstable and none semistable. Two stable and two semistable. Two unstable and two semistable.

For what value of the constant b will the following differential equation be exact?

$$(ye^{3xy} - x)dx + bxe^{3xy}dy = 0$$

1 3 -1 0

No value of b. The water bath surrounding a certain nuclear reactor contains radioactive material that decays with a half-life of 5 days. New radioactive material is added to the bath at the constant rate of 1 gram per day. Assume the bath contains no radioactive material initially. Find an expression for the amount $Q(t)$ of grams of radioactive material after t days. $Q(t) = \frac{5}{\ln 2}(1-(\frac{1}{2})^{\frac{t}{5}})$ $Q(t) = \frac{5}{\ln 2}((\frac{1}{2})^{\frac{t}{5}} - 1)$ $Q(t) = 5\ln 2((\frac{1}{2})^{\frac{t}{5}} - 1)$ $Q(t) = 5\ln 2(1 - (\frac{1}{2})^{\frac{t}{5}})$ $Q(t) = (1 - (\frac{1}{2})^{\frac{t}{5}})$

The equation $y' = \frac{x^2-14xy+3y^2}{7x^2+y^2} - 1$ is homogeneous. linear. separable but not exact. exact with a suitable choice of integrating factor. none of the others.

A particular solution of $y'' + y = t\cos t$ has the form $(At^2 + Bt)\cos t + (Ct^2 + Dt)\sin t$. $(At + B)\cos t + (Ct + D)\sin t$. $(At + B)\cos t + (At + B)\sin t$. $At^2\cos t + Bt^2\sin t$. $(At + B)\cos t$

The general solution of the equation $y'' + y = \csc t$, for $0 < t < \pi$ is $c_1 \cos t + c_2 \sin t - t \cos t + (\sin t) \ln \sin t$. $c_1 \sin t + c_2 \cos t + t \sin t - (\cos t) \ln \sin t$. $c_1 e^t \sec t + c_2 e^t \csc t$. $c_1 t \sin t + c_2 (\cos t) \ln \sin t$. $(c_1 + t) \cos t + (c_2 + \ln \cos t) \sin t$.

Find $y(1)$ if $y'' + 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = 0$. 0.301 0.187 0.241 0.282 0.666

For $x > 0$ the Wronskian of two independent solutions of the Euler equation $x^2 y'' + xy' - y = 0$ is a constant multiple of $\frac{1}{x}$. x. $\frac{1}{x^2}$. x^2. 1.

Find $y(1)$ if $y(t)$ is the solution of the initial value problem

$$y'' - y' = -2t, \quad y(0) = 0, \quad y'(0) = 2.$$

3 $4 + e$ 4 $e + 2$ 1

Find the first three nonzero terms of a power series solution to the initial value problem

$$(1 - x^2)y'' + 3y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

$x - \frac{1}{2}x^3 - \frac{3}{40}x^5$ $1 + \frac{1}{2}x^3 - \frac{3}{8}x^5$ $x - \frac{1}{2}x^3 + \frac{3}{20}x^5$ $1 + x + \frac{1}{3}x^3$ $x - \frac{1}{6}x^3 - \frac{1}{40}x^5$

The equation $(x^2 + 16)y'' + 3y' - 4x^2 y = 0$ has a series solution in powers of $x - 3$. From the general theory, its radius of convergence ρ must satisfy $\rho \geq 5$. $\rho \geq 4$. $\rho \geq 3$. $\rho \geq 16$. $\rho = \infty$.

Consider the equation $x(1 - x)y'' + (2 + x)y' + y = 0$. For which values of r is there sure to be a solution of the form $|x - 1|^r \sum_{n=0}^{\infty} a_n (x - 1)^n$, with $a_0 \neq 0$? 4 3 2 0 No value of r.

Which of the following statements about the matrix A are true?

$$A = \begin{bmatrix} 2 & 4 & 2 & -1 & 0 & 2 & -2 \\ 4 & 8 & 4 & -2 & 1 & 7 & -4 \\ 0 & 0 & 0 & 2 & 0 & 4 & 4 \end{bmatrix}$$

A. The first row of the reduced echelon form of A is $[1 \quad 2 \quad 1 \quad 0 \quad 0 \quad 2 \quad 0]$.

B. The system $Ax = b$, where x is 7×1 and b is 3×1, has a solution for any choice of b.

C. The dimension of the column space of A is 3.

D. The dimension of the solution space of A is 3.

All except D. All except C. All except B. All except A. All are true.

If $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, which row of A^{-1} is orthogonal to the other two rows of A^{-1}? Only the first row. Only the second row. Only the third row. All the rows. None of the rows.

The vectors $v_1 = (1, 0, 1, 1)$, $v_2 = (2, 1, 1, 4)$, $v_3 = (0, 2, 3, 4)$ are linearly independent and do not span $\mathbf{R}^4$. are linearly independent and span $\mathbf{R}^4$. are linearly dependent and do not span $\mathbf{R}^4$. are linearly dependent and span $\mathbf{R}^4$. are linearly independent and span a two-dimensional subspace of $\mathbf{R}^4$.

Let A be an $m \times n$ matrix of rank r. Suppose the row space and the solution space of A have the same dimension, and that $Ax = b$ always has a solution. Then $n = 2m$. $n = m$. $n = r = 2m$. $n + r = m$. $n = r = m$.

Find $|A|$ if A is the matrix of the linear transformation L defined by

$$L \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3x_1 + 5x_2 - 2x_3 + 6x_4 \\ x_1 + 2x_2 - x_3 + x_4 \\ 2x_1 + 4x_2 + x_3 + 5x_4 \\ 3x_1 + 7x_2 + 5x_3 + 3x_4 \end{bmatrix}$$

-18 -36 -9 -27 0

In the system below, if the coefficient matrix has determinant 25, find x_1.

$$\begin{bmatrix} 11 & 0 & 2 & 0 & 3 \\ 2 & 4 & 0 & 5 & 0 \\ -1 & 0 & 7 & 0 & 8 \\ 14 & 9 & 0 & 10 & 0 \\ -12 & 0 & 12 & 0 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \\ 0 \\ 11 \end{bmatrix}$$

0 $\frac{4}{5}$ $-\frac{2}{5}$ $\frac{3}{5}$ $-\frac{1}{5}$

Find the entry in the fourth row and second column of A^{-1} if

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}.$$

$-\frac{1}{8}$ 4 0 $\frac{1}{4}$ $-\frac{1}{16}$

Find the polynomial whose roots are the eigenvalues of the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$. $(3-\lambda)^2(5-\lambda)$ $(3-\lambda)(5-\lambda)^2$ $(3-\lambda)(4-\lambda)^2$ $(3-\lambda)((4-\lambda)^2+1)$ $15\lambda - 8\lambda^2 + \lambda^3$

Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Then A has 2 as an eigenvalue, and -1 as an eigenvalue of multiplicity two. Use **reduced** echelon form to find the eigenvectors of A. For which of the following matrices P will $P^{-1}AP$ be a diagonal matrix? $P = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $P = \begin{bmatrix} -2 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \; P = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$