Math 228, Test 2

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Instructions: You have one hour for the exam. There are 11 problems, worth 9 points each, with one free point, for a total of 100 points possible. For multiple-choice problems, please mark your answer clearly. For all other problems, please *show your work completely.* Partial credit will be given for all non-multiple-choice problems.
Calculators, notes, and books are prohibited.
This exam is bound by the provisions of the Notre Dame Honor Code.

Name:

1. Do the vectors $v_1 = (1, 2, 0)$, $v_2 = (0, 1, 2)$ and $v_3 = (1, 1, 1)$ constitute a basis for $\mathbb{R}^3$? Show your work and explain clearly why or why not.

2. Let u and v be two vectors in $\mathbb{R}^n$ with $\|$ u$=5$ and $\|v\| = 6$. What is the largest possible value for $|u \cdot v|$? What is the largest possible value for $\|u + v\|$? Explain your reasoning.

3. Suppose $S = \{v_1, v_2, v_3\}$ is a linearly *dependent* set of vectors in $\mathbb{R}^3$. Which of the following statements are necessarily true?

(I) The span of S is a plane through the origin
(II) The dimension of the span of S is at most 2
(III) The span of S cannot be all of $\mathbb{R}^3$

a) None of them b) I and II only c) I only d) II and III only e) I, II, and III

4. Let A be the matrix

A=(
[c]rrrr
1 2 1 0
3 0 1 2
1 -4 -1 2
.

Find a basis for $RS(A)$, a basis for $NS(A)$,
and a basis for $CS(A)$. State the dimensions of $RS($
A, $NS(A)$, and $CS(A)$.

5. Suppose U is a matrix in row-echelon form obtained from a matrix A by means of row operations. Which of the following statements are necessarily true?

(I) $NS\,(A) = NS(U)$

(II) $RS(A) = RS(U)$

(III) $CS(A) = CS(U)$

(IV) $rk\,(A) = rk\,(U)$

(V) $\dim CS(A) = \dim CS(U)$

a) All of them b) IV and V only c) I, II, and IV only

$d) I, II, IV, and V only$ $e) I and II$ *only*

6. Below are several examples of a vector space V and a subset S of V. In each case state whether or not S is a subspace of V. If S is *not* a subspace of V, explain why not.

a) $V = C[0,5]$, $S = \{f \in V | f(0)$
$=3$

b) $V = M_{33}$, $S = \{A|$ A is invertible$\}$

c) $V = \mathbb{R}^3$, $S = \{(0,t,2t)|$
t$\in \mathbb{R}$

d) $V = C[a,b]$, $S = \{f \in V | f(x)$
≤ 0 for all x

7. Determine whether the following polynomials constitute a basis for $\mathcal{P}_2$:

$\mathrm{p}_1(x) = 1$

$\mathrm{p}_2(x) = 1 + x$

$\mathrm{p}_3(x) = 1 + x + x^2$.

If they do, find the coordinates of $f(x) = 3 + 4x + 5x^2$ with respect to this basis.

8. Consider the vectors $u_1 = (1, 1, 0, 0)$, $u_2 = ($
1,0,2,1 and $u_3 = (1, 2, -2, -1)$ in $\mathbb{R}^4$.
What is the dimension of the span of $\{u_1, u_2, u_3\}$?
Show your work and explain your reasoning clearly. *Hint*: are the vectors linearly independent?

9. Determine whether each of the following collections of vectors in linearly independent. Explain your reasoning in each case.
   a) $u_1 = \left(2, \frac{3}{2}, 50, \frac{5}{3}\right)$, $u_2 = ($
   $23{,}5\overline{_{6{,}105{,}30{,}}}$ $u_3 = \left(\frac{3}{7}, 0, 22, -11\right)$
   ,$u_4 = (22, 0, 0, 9)$, $u_5 = (1, 1, 1, 0, -99)$ in
   $\mathbb{R}^4$
   b) $w_1 = (1, 2, 5, 6, 7)$, $w_2 = (7, 6, 5, 2, 1)$,
   $w_3 = (0, 0, 0, 0, 0)$ in $\mathbb{R}^5$

10. Consider the linear transformation $T : \mathbb{R}^3 \rightarrow$ R^2 given by

$$\mathrm{T}(x, y, z) = (3x - z, 2y + z)\,.$$

Find the matrix that represents T. (That is, find the matrix A such that $f_A = T$.)

11. Suppose that A is a 3×4 matrix and that $\dim NS$ (A= 1. Which of the following statements are necessarily true?

(I) The rows of A are linearly independent
(II) The columns of A are linearly dependent
(III) The row echelon form of A has a row of all zeros
(IV) The rank of A is one

a) None of them b) All of them c) I and II only d) I, II, and III only e) II only