Math 228, Test 3

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Instructions: You have one hour for the exam. There are 10 problems, worth 10 points each for a total of 100 points possible. For multiple-choice problems, please mark your answer clearly. For all other problems, please *show your work completely.* Partial credit will be given for all non-multiple-choice problems.
Calculators, notes, and books are prohibited.
This exam is bound by the provisions of the Notre Dame Honor Code.

Name:

1. Apply the Gram-Schmidt procedure to the vectors $v_1 = ($
1,0,0, $v_2 = (1, 2, -1)$, $v_3 = (0, 1, 1)$
$toobtainanorthonormalsetofvectors$u$_1$, u_2, u_3.

2. Find all the eigenvalues and eigenvectors of the matrix

A=[

[c]rr

3 1

-1 1

.

Is A diagonalizable? Explain clearly why or why not.

3. Suppose that Q is an $n \times n$ orthogonal matrix. Which of the following statements are always true?
(I) $Q^{-1} = Q^T$
(II) The rows of Q are orthogonal
(III) Q^{-1} is orthogonal

(a) None of them (b) I only (c) I and II only (d) I and III only
(e) I, II, and III

4. Set up but *do not solve* the normal equations for finding the best fit line to the points $(-1, 0)$, $(0, 0)$, $(1, 2)$, $(2, 2)$. ("Best fit" means in the least squares sense.) You do *not* have to compute the product of the matrices involved.

5. Consider the matrix

A=[

[c]ccc

2 1 1

1 2 1

1 1 2

.

Find an orthonormal basis for $\mathbb{R}^3$ consisting of eigenvectors for A.
Hint: The characteristic polynomial of A is $(4-\lambda$
$(\lambda-1)^2$.

6. Suppose that A is the 2×2 matrix given by

A=SΛS^{-1}

where

S=[
[c]rr
1 2
0 -1

and

Λ = [
[c]cc
2 0
0 0
.

Compute A^6.

7. Let Q be an orthogonal $n \times n$ matrix, and let $A = Q^3$. Is A necessarily orthogonal? Explain why or why not.
Hint: Compute $A^T A$.

8. Let A be the matrix

A=[

[c]ccc

$1\frac{}{\sqrt{2}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{6}}}$

0 $1\frac{}{\sqrt{3}-\frac{2}{\sqrt{6}}}$

-1$\frac{}{\sqrt{2}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{6}}}$

.

a) Show that A is orthogonal.

b) Compute A^{-1}.

9. Let A be an $n \times n$ matrix. For each of the following statements, circle "True" if the statement is necessarily true, and "False" if the statement could be false.

(I) (True or False) If A is diagonalizable, then A has n distinct eigenvalues.

(II) (True or False) If A is invertible then zero is not an eigenvalue of A.

(III) (True or False) If zero is not an eigenvalue of A then A is invertible.

(IV) (True or False) If A is symmetric then A has n distinct eigenvalues.

(V) (True or False) If A is symmetric then A is diagonalizable.

10. a) Suppose that A is a 2×2 symmetric matrix whose characteristic polynomial is $\lambda^2 - 2\lambda + 1$. Show that $A = I$.

b) Give an example of a 2×2 matrix B whose characteristic polynomial is $\lambda^2 - 2\lambda + 1$ but such that $B \neq I$.