Math 228, Test 1

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Instructions: You have one hour for the exam. There are 10 problems worth 10 points each for a total of 100 points possible. On all problems except Problem 6, please *show your work completely* and partial credit will be given. For Problem 6, each item is worth two points, with no partial credit within items.
Calculators, notes, and books are prohibited.
This exam is bound by the provisions of the Notre Dame Honor Code.

Name:

1. Suppose that E is an $n \times n$ elementary matrix corresponding to the operation of interchanging two rows. What is E^{-1}? What is E^{37}? Explain.

2. Let A be the matrix

A=[

[c]rrrr

0 3 1 18

0 -9 1 8

0 0 0 2

2 22 3 -14

.

Find the entry in the second row and the first column of A^{-1}.
Hint: Use the formula for A^{-1} in terms of $adj(A)$.

3. Compute the value of the following determinant.

det [

[c]ccc

a b c

d e f

a+2d b+2e c+2f

Explain your reasoning clearly.

4. Find the L-U decomposition of the matrix

A=[

[c]rrr

2 1 3

-1 0 2

2 2 -3

.

No row interchanges should be needed.

5. Suppose that A and B are $n \times n$ invertible matrices. Simplify the following expression:

$$\left(A(BA)^{-1}B(AB)^{-1}A^2\right)^{-1}.$$

6. Suppose that A and B are invertible $n \times n$ matrices. Mark each of the following statements as **T** (always true) or **F** (not always true).

---- $(AB)^{-1} = A^{-1}B^{-1}$

---- $\det\left((AB)^{-1}\right) = \det($

A$^{-1}B^{-1}$

---- $\det A > 0$

---- $\det(2A) = 2\det A$

---- $\det(I + A) = 1 + \det A$

7. Find the general solution of the system of equations represented by the following augmented matrix:

[

[c]rrrr

2 1 4 0

2 2 5 1

2 -1 2 -2

.

8. Suppose A is a 2×2 matrix and that $A^2 = 0$. (Here 0 means the zero matrix, all of whose entries are zero.) Is it necessarily true that $A = 0$? If so, explain why. If not, give an example of a 2×2 matrix A such that $A^2 = 0$ but $A \neq 0$.

9. What is the value of the following determinant?

det [

[c]rrrr

1 2 57 -9

2 1 8 43

0 0 -1 1

0 0 4 1

10. Compute A^{-1} if A is the matrix

A=[
[c]rrr
1 2 2
0 2 1
0 -2 1
.