MATH 228: Intro to Lin Alg & Diff Eqns Name:____________________________

Exam I *February 12, 2002* Instructor:____________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 10 multiple choice questions worth 5 points each and 3 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems *you must show your work and all important steps to receive credit.*

The augmented matrix of a system of equations has been reduced to row echelon form:

$$\begin{bmatrix} 2 & -1 & 5 & -3 & 1 \\ 0 & 1 & -3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find all solutions of the system.
$\{(-s+t+1, 3s-t+1, s, t) \mid s, t \text{ real numbers}\}$ $\{(-s+t+1, 3s-t+1, 0, 0) \mid s, t \text{ real numbers}\}$
$x_1 = 1,\ x_2 = 3,\ x_3 = 1,\ x_4 = 1$ $\{(-r+s-t, 3r-s-t+1, r, s, t) \mid r, s, t \text{ real numbers}\}$
There are no solutions.

Let $A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -3 & 0 & 3 & -8 & -10 \\ 1 & 0 & -1 & 3 & 4 \\ 2 & 0 & -2 & 5 & 6 \end{bmatrix}$. Determine which of the following matrices gives A reduced to row echelon form.

$\begin{bmatrix} 1 & 0 & -1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 1 & 3 & 4 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 3 & -2 & 6 \\ 2 & 0 & 4 & 7 \end{bmatrix}$. Determine which of the following statements is true.

If $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, then AB is defined and is a 2×1 matrix.

If $B = \begin{bmatrix} 1 & 2 & 3 & 0 \\ -1 & 1 & 2 & 4 \end{bmatrix}$, then AB is defined and is a 2×4 matrix.

If $B = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$, then AB is defined and is a 2×4 matrix.

If $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, then AB is defined and is a 4×1 matrix.

If $B = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 3 & 2 \\ 4 & 0 \end{bmatrix}$, then AB is defined and is a 4×2 matrix.

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$. Find A^{-1}.

$A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ -4 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -1 & 0 & -1 \\ 3 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 3 & 1 & -1 \\ -4 & -1 & 2 \\ 2 & 0 & -1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & -1 \end{bmatrix}$

A is not invertible.

Let A be an $n \times n$ matrix. Which of the following statements are NOT correct?

(I) $\det A = \det A^T$

(II) $\det(AB) = (\det A)(\det B)$

(III) $\det(A + B) = \det A + \det B$

(IV) $\det(2A) = 2^n \det A$

(V) If A is not invertible, then $\det A = 0$.

(III) only (V) only (IV) only (III) and (IV) (I) and (V)

Compute $\det \begin{bmatrix} 1 & 2 & 1 & -1 \\ -1 & -2 & -1 & 2 \\ 2 & 4 & 1 & -2 \\ 3 & 2 & 3 & -3 \end{bmatrix}$.

-4 0 -1 6 4

Find the $(2, 3)$-cofactor of $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 4 & -3 \\ 5 & 3 & 9 \end{bmatrix}$.

-13 3 -7 18 -6

Let $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. If X is the solution of $AX = B$, determine which of the following expressions gives x_3, according to Cramer's Rule.

$\frac{1}{4} \det \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ $\frac{1}{4} \det \begin{bmatrix} 2 & 1 & 0 \\ -1 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix}$ $\frac{1}{4} \det \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix}$ $\frac{1}{4} \det \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$\frac{1}{4} \det \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

Let $\mathbf{u} = (2, -2, 0, 0)$, and $\mathbf{v} = (3, 0, 0, -3)$. Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

$\pi/3$ $\pi/2$ $\pi/4$ $\pi/6$ $3\pi/4$

Let $\mathbf{u} = (1, 1, 0)$, $\mathbf{v} = (1, 0, -1)$, and $\mathbf{w} = (-1, -1, 0)$. Determine which of the following describes Span $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.

point line plane R^3 none of the above

11. Find all solutions of the system

$$\begin{aligned}
x_1 + 2x_2 - x_3 &= 3 \\
2x_1 + 4x_2 - x_3 &= 4 \\
x_1 + 3x_2 &= 5 \\
-3x_1 - 5x_2 + 3x_3 &= -5
\end{aligned}$$

12. Find the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 0 \\ 2 & 11 & 5 \end{bmatrix}$.

13. Let V be the vector space of polynomials of degree ≤ 2 with inner product defined for $f, g \in V$ by

$$f \cdot g = f(0)g(0) + f'(0)g'(0) + \frac{1}{4}f''(0)g''(0)$$

Compute the projection of $f(x) = 1 + x + x^2$ on $g(x) = x^2 - 2$.