MATH 228: Intro to Lin Alg and Diff Eqns Name:______________________________

Exam II *March 19, 2002* Instructor:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 10 multiple choice questions worth 5 points each and 3 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems *you must show your work and all important steps to receive credit.*

Let $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (1, 0, 1)$, $\mathbf{v}_3 = (-1, c, -3)$. Find a value of c such that $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are linearly dependent.
2 0 -2 1 -3
Determine which of the following sets of polynomials forms a basis of P_2, the vector space of all polynomials of degree less than or equal to 2.
$\{1+x-x^2, x+x^2, 1+x+x^2\}$ $\{1+71x+23x^2, 2x+59x^2\}$ $\{1+2x+x^2, 2-x+x^2, 3+x+2x^2\}$ $\{1+x, 1+2x+x^2, 3+4x+5x^2, 8+7x+2x^2\}$ $\{3-x+x^2, 5x+x^2, 1+2x+3x^2, 1+4x+5x^2\}$
What is the dimension of the space spanned by $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$.
2 3 4 1 5

Determine the rank of the matrix $A = \begin{bmatrix} 2 & 6 & -1 & 1 \\ -1 & -3 & 1 & 0 \\ -3 & -9 & 2 & -1 \\ 4 & 12 & -2 & 2 \end{bmatrix}$.
2 0 1 3 4
Determine which of the following sets are vector spaces with usual addition and scalar multiplication.
(I) Set of all continuous functions f on $[0, 1]$ with $f(0) = 0$ and $f(1) = 1$.
(II) Set of all 2×2 matrices of the form $\begin{bmatrix} 0 & a \\ b & c \end{bmatrix}$ where a, b, c are arbitrary real numbers.
(III) Set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 0 & b \end{bmatrix}$ where a, b are arbitrary real numbers.
(IV) Set of all polynomials $p(x) = a_0 + a_1x + a_2x^2$ with $a_0 + a_2 = 0$.
(V) Set of all 2×2 matrices A with $\det A = 0$.
(II) and (IV) (I) and (II) (IV) and (V) (I) and (III) (II), (IV) and (V)
Find the coordinates, $[\mathbf{x}]_B$ of the vector $\mathbf{x} = (1, 0, -1)$ with respect to the basis $B = \{(1, -1, 0), (-1, 1, -1), (0, -1, 1)\}$. $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$
Suppose $B = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $C = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are bases of a vector space V related by

$$\mathbf{u}_1 = \mathbf{v}_1 - \mathbf{v}_3$$
$$\mathbf{u}_2 = \mathbf{v}_2 - \mathbf{v}_3$$
$$\mathbf{u}_3 = \mathbf{v}_1 - \mathbf{v}_2$$

If $[x]_B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, find $[x]_C$.

$\begin{bmatrix} 4 \\ -1 \\ -3 \end{bmatrix}$ $\begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$

The Gram-Schmidt process is applied to the following basis of $\mathbf{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}$$

resulting in the orthonormal vectors

$$\mathbf{q}_1 = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}, \quad \mathbf{q}_2 = \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \quad \mathbf{q}_3$$

Find $\mathbf{q}_3$.

$\begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix}$ $\begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$ $\begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1/3 \\ 2/3 \\ -1/3 \end{bmatrix}$

Let $V = C[0,1]$ be the vector space of continuous functions on $[0,1]$ with inner product defined by $f \cdot g = \int_0^1 f(x)g(x)\,dx$. Let $f(x) = -6x + 2$. Find the number a such that $g(x) = x + a$ is orthogonal to f.

-1 0 -3 2 $1/3$

Find R in the QR-decomposition of the matrix $A = \begin{bmatrix} 4 & 25 \\ 0 & 0 \\ 3 & -25 \end{bmatrix}$. $\begin{bmatrix} 5 & 5 \\ 0 & 35 \end{bmatrix}$ $\begin{bmatrix} 5 & 0 \\ 15 & 35 \end{bmatrix}$

$\begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}$ $\begin{bmatrix} 4 & 3 \\ 0 & 25 \end{bmatrix}$ $\begin{bmatrix} 12 & -4 \\ 0 & 25 \end{bmatrix}$

11. Find a basis for the null space, $NS(A)$, of the matrix $A = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & -1 & 2 & 1 \\ -1 & 0 & -1 & 0 \\ 3 & -2 & 3 & 2 \end{bmatrix}$.

12. Define $T : C[0,1] \to C[0,1]$ as follows: If f is a continuous function on $[0,1]$, then $T(f)$ is the continuous function of x defined by $\int_0^x tf(t)\,dt$ for $0 \leq x \leq 1$. For example, if f is the function $f(x) = x^2$, then $T(f)$ is the function $\int_0^x t \cdot t^2\,dt = x^4/4$. Determine whether T is a linear transformation. Be sure to justify the statements leading to your conclusion.

13. Let $A = \begin{bmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{bmatrix}$.

(a) Find a scalar c so that cA is an orthogonal matrix.

(b) Use your answer in part (a) to find A^{-1}.