MATH 228: Intro to Lin Alg and Diff Eqns Name:______________________________

Exam III *April 23, 2002* Instructor:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 10 multiple choice questions worth 5 points each and 3 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems *you must show your work and all important steps to receive credit.*

You may *not* use a calculator.

Determine which of the following matrices is similar to $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$.

$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Determine which of the following vectors is an eigenvector of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

The vectors $(1,1,0)$, $(1,0,-1)$, $(1,-1,1)$ are eigenvectors of a symmetric matrix A. Find an orthogonal matrix Q such that $Q^T AQ$ is a diagonal matrix.

$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{6} & -1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{bmatrix}$ $Q = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 \\ 0 & 1/\sqrt{3} & 0 \\ 0 & 0 & -1/\sqrt{6} \end{bmatrix}$ $Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{3} \\ 1/\sqrt{2} & 0 & 1/\sqrt{3} \\ 0 & -1/\sqrt{2} & -1/\sqrt{3} \end{bmatrix}$

$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$ $Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & -1/\sqrt{6} \\ 0 & 1/\sqrt{3} & 2/\sqrt{6} \end{bmatrix}$

Find the general solution of the differential equation $y' + \tan(t)y = \cos(t)$.

$y = (t+c)\cos(t)$ $y = t\cos(t) + c$ $y = \frac{t+c}{\cos(t)}$ $y = t\sin(t) + c$ $y = \frac{\frac{1}{2}t + \frac{1}{4}\sin(2t) + c}{\cos(t)}$

Solve the initial value problem $y' = (1+y)^2 e^{-x}$, $y(0) = 0$.

$y = e^x - 1$ $y = e^{-x} - 1$ $y = e^{x-1}$ $y = \frac{1}{3}e^{x/3} - 1$ $y = \left(e^{-x} - \frac{1}{2}\right)^{-1} - 1$

The population of moths in a certain area increases at a rate proprtional to the current population and, in the absence of other factors, the population doubles every week. However, birds in the area eat 10 moths per day. If $P(t)$ denotes the number of moths after t weeks, determine which of the following equations describes $P(t)$.

$\frac{dP}{dt} = \ln(2)P - 70$ $\frac{dP}{dt} = 2P - 70$ $\frac{dP}{dt} = 2P - 10$ $\frac{dP}{dt} = 2t - 70$ $\frac{dP}{dt} = \ln(2)P - 10t$

Solve the initial value problem $y'' + 2y' - 15y = 0$, $y(0) = 1$, $y'(0) = 2$.

$\frac{7}{8}e^{3t} + \frac{1}{8}e^{-5t}$ $\frac{3}{2}e^{3t} - \frac{1}{2}e^{5t}$ $\frac{1}{3}e^{3t} + \frac{2}{3}e^{-5t}$ $\frac{3}{8}e^{-3t} + \frac{5}{8}e^{5t}$ $\frac{1}{2}e^{2t} + \frac{1}{2}e^{-15t}$

Find the general solution to the differential equation $y'' + 6y' + 10y = 0$.

$c_1e^{-3t}\cos(t) + c_2e^{-3t}\sin(t)$ $c_1e^{3t}\cos(t) + c_2e^{3t}\sin(t)$ $c_1e^{t}\cos(3t) + c_2e^{t}\sin(3t)$ $c_1e^{-3t} + c_2e^{t}$ $c_1e^{6t} + c_2e^{10t}$

Let y_1 and y_2 be solutions of $y'' + p(t)y' + q(t)y = 0$ where p and q are continuous on an open interval I. Four of the following statements are equivalent, in the sense that each one implies the other three. Find the one statement that is *not* equivalent to any of the other four.
$y_1(t) \neq y_2(t)$ for all t in I. y_1, y_2 are a fundamental set of solutions on I. y_1, y_2 are linearly independent on I. $W(y_1, y_2)(t_0) \neq 0$ for some t_0 in I. $W(y_1, y_2)(t) \neq 0$ for all t in I.
Determine the longest interval in which the following initial value problem is certain to have a unique twice differentiable solution.

$$(t^2 + t - 6)y'' + \cos(t)y' - \ln(t^2)y = e^t, \quad y(-1) = 1, \quad y'(-1) = 2$$

$(-3, 0)$ $(0, 2)$ $(-3, 2)$ $(-\infty, 0)$ $(-2, \infty)$

11. Find the matrix of the linear transformation $T(x, y) = (3x - y, 2x + 2y)$ with respect to the basis $B = \{(5, 2), (2, 1)\}$.

12. Consider the differential equation $(2 + \sin(y))dx + \cos(y)dy = 0$.

(a) Show that the equation *not* exact.

(b) Find an integrating factor, $\mu = \mu(x)$, for the equation.

(c) Use your answer in part (b) to solve the equation.

13. Determine whether the functions $y_1(x) = x$ and $y_2(x) = x \cos x$ are linearly independent. Justify your answer.