1. Mark each of the following statements as either true or false:

a) An $n \times n$ symmetric matrix always has n different eigenvalues.

b) An $n \times n$ symmetric matrix always has n linearly independent eigenvectors.

c) Two different eigenvectors of a symmetric matrix are always orthogonal.

d) A symmetric matrix can always be diagonalized.

2. The matrix $A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ has eigenvalues $\lambda_1 = 0, \lambda_2 = 3, \lambda_3 = 5$. Find an orthogonal

matrix Q so that $Q^T A Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$