Name:______________________________

Instructor:___________________________

MATH 228: Introduction to Linear Algebra and Differential Equations

Exam I

February 21, 2003

There are 20 problems on pages 2–11. Each problem is worth 5 points. You must work on the exam entirely on your own. You are not allowed to discuss the exam with anyone until after they are turned in. You may use your textbook and a calculator. *To receive credit, you must show all your work and include all important steps and explanations. A correct answer without supporting work will receive no credit.*

Please work out the problems on separate sheets and then copy your work and answers neatly into the space below the appropriate problem. You will lose credit if your answers are difficult to read or your work is poorly organized.

	Score
1.	______
2.	______
3.	______
4.	______
5.	______
6.	______
7.	______
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10.	______
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12.	______
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16.	______
17.	______
18.	______
19.	______
20.	______
Total:	______

1. Solve the system by Gaussian elimination.

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & - & x_3 & + & 2x_4 & = & 2 \\ 3x_1 & + & 5x_2 & + & x_3 & - & 2x_4 & = & -8 \\ -4x_1 & - & 7x_2 & - & x_3 & + & x_4 & = & 13 \end{array}$$

2. Form all possible matrix products of the following matrices.

$$A = \begin{bmatrix} 1 & 2 \\ 4 & -1 \\ 0 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & -1 & 2 \end{bmatrix}$$

3. Let

$$A = \begin{bmatrix} 1 & 4 & 6 & 3 \\ 0 & 0 & 1 & 2 \\ 3 & 10 & 9 & 10 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 4 & 6 & 3 \\ 0 & -2 & -9 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Find two elementary matrices, E_1, E_2, such that $E_1E_2A = B$.

4. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ k & 1 & 0 & 0 \\ 0 & k & 1 & 0 \\ 0 & 0 & k & 1 \end{bmatrix}$.

5. Evaluate the determinant by reducing the matrix to row-echelon form.

$$A = \begin{bmatrix} 0 & 1 & 3 & 4 \\ 2 & -7 & 2 & 4 \\ -3 & 11 & 4 & 0 \\ 1 & -3 & 2 & 2 \end{bmatrix}$$

6. Calculate $\det(3A^{-2})$ where A is the matrix in problem 5.

7. Evaluate the determinant by cofactor expansion.

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix}$$

8. Find the standard matrix for the linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ that rotates counterclockwise about the x-axis by $45°$, then reflects about the yz-plane.

9. If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^n$ with $\mathbf{u} \cdot \mathbf{v} = 2$ and $\|\mathbf{u} + \mathbf{v}\| = 3$, calculate $\|\mathbf{u} - \mathbf{v}\|$.

10. Show that the range of the linear operator defined by

$$\begin{array}{rcrcrcr} w_1 & = & x_1 & + & 4x_2 & - & 3x_3 \\ w_2 & = & 2x_1 & + & 6x_2 & + & 5x_3 \\ w_3 & = & x_1 & + & 6x_2 & - & 14x_3 \end{array}$$

is not R^3, and find a vector that is not in the range.

11. Let V be the set of all vector functions $\mathbf{r}(t) = (x(t), y(t))$ with the usual operations of addition and scalar multiplication (e.g., $(t, t^2) + (t^3, t - 1) = (t + t^3, t^2 + t - 1)$ and $3(t, t^2) = (3t, 3t^2)$). Determine whether V is a vector space.

12. Determine whether the set of 2×2 matrices A with $\det A = 0$ is a subspace of the vector space of all 2×2 matrices.

13. Determine the values of x, if any, for which the set of vectors $\{(1, x, x), (x, 1, x), (x, x, 1)\}$ is linearly dependent.

14. Find the coordinate vector of $1 + 3x + 5x^2$ relative to the basis $\{1 + x + x^2, 1 - x + x^2, 1 - x - x^2\}$ of P_2.

15. Find a basis for the nullspace, row space, and column space of the matrix $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 2 & 2 & 0 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix}$.

16. Determine whether $\mathbf{w} = \begin{bmatrix} 11 & -1 \\ -1 & 2 \end{bmatrix}$ is in the span of

$$\mathbf{v}_1 = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 & 3 \\ 3 & 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$$

and, if it is, express $\mathbf{w}$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

17. Determine whether the set of vectors in $\mathbb{R}^4$, $\{(3, 1, 1, 2), (-4, 0, 1, 1), (1, 2, -1, 3), (-1, -4, 6, -4)\}$, is linearly independent.

18. Use the Wronskian to determine whether the set $\{x^2, 1 + x^2, x^3\}$ is linearly independent.

19. Find the eigenvalues and eigenvectors of the linear operator $T(x, y) = (2x + 2y, 5y - x)$.

20. Use the information in the table to determine whether the linear system $A\mathbf{x} = \mathbf{b}$ is consistent, and, if so, determine the number of parameters in its general solution.

	size of A	rank(A)	rank$[A\|\mathbf{b}]$
a)	4×3	3	3
b)	3×5	3	4
c)	5×4	3	3
d)	6×9	5	5
e)	2×7	1	2