1. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 26 \\ -13 \\ 0 \end{bmatrix}$. Determine the least squares solution of the linear system $A\mathbf{x} = \mathbf{b}$.

(a) $\begin{bmatrix} 7 \\ -3 \end{bmatrix}$ (b) $\begin{bmatrix} 11 \\ 7 \\ 5 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 12 \\ 10 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 7 \end{bmatrix}$ (e) $\begin{bmatrix} 5 \\ -11 \end{bmatrix}$

2. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$. Find $A \cdot B$.

(a) $\begin{bmatrix} 4 & -1 \\ 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 2 & 4 \\ 4 & 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 4 & 4 \\ -3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ (e) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \\ 2 & 3 \end{bmatrix}$

3. Let $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find A^{-1}.

(a) $\begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1/3 & 1/2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/3 & -1/2 \\ 0 & -1 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Compute the determinant of $\begin{bmatrix} 4 & 0 & 3 \\ 2 & 2 & -1 \\ 5 & 0 & 6 \end{bmatrix}$.

(a) 18 (b) 48 (c) 8 (d) -30 (e) 60

5. Let $A = \begin{bmatrix} 6 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$. If $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is a solution of $A \cdot X = B$, determine which of the following gives x_1.

(a) $x_1 = \dfrac{\det \begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 6 & 1 & 0 \end{bmatrix}}{\det \begin{bmatrix} 6 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}}$ (b) $x_1 = \dfrac{\det \begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 6 & 1 & 0 \end{bmatrix}}{\det \begin{bmatrix} 1 & 6 & 3 \\ 3 & 3 & 2 \\ 6 & 1 & 0 \end{bmatrix}}$ (c) $x_1 = \dfrac{\det \begin{bmatrix} 6 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}}{\det \begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 6 & 1 & 0 \end{bmatrix}}$

(d) $x_1 = \dfrac{\det \begin{bmatrix} 1 & 6 & 3 \\ 3 & 3 & 2 \\ 6 & 2 & 1 \end{bmatrix}}{\det \begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 6 & 1 & 0 \end{bmatrix}}$ (e) $x_1 = \dfrac{\det \begin{bmatrix} 6 & 3 & 1 \\ 3 & 2 & 3 \\ 2 & 1 & 6 \end{bmatrix}}{\det \begin{bmatrix} 6 & 3 & 1 \\ 3 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}}$

6. Determine which of the following matrices gives R in the QR decomposition of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

(a) $\begin{bmatrix} \sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & \sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \sqrt{2} & 0 & 1/\sqrt{3} \\ 0 & \sqrt{2} & 2/\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} \sqrt{2} & 0 & -1/\sqrt{6} \\ 0 & \sqrt{2} & 1/\sqrt{6} \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} \sqrt{2} & 0 & 1/\sqrt{3} \\ 0 & \sqrt{2} & 1/\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} \sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & \sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} \end{bmatrix}$

7. Determine which of the following statements is *not* equivalent to A being an $n \times n$ orthogonal matrix.

(a) $A\mathbf{v}$ is orthogonal to $\mathbf{v}$ for all vectors $\mathbf{v}$ in R^n. (b) The column vectors of A are orthonormal.

(c) The row vectors of A are orthonormal. (d) $A^{-1} = A^T$

(e) $\|A\mathbf{v}\| = \|\mathbf{v}\|$ for all vectors $\mathbf{v}$ in R^n.

8. A matrix A has eigenvectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ with corresponding eigenvalues 2, -1, and -2. Use this information to reconstruct the matrix A.

(a) $\begin{bmatrix} 2 & -3 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & 0 & -2 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & -3 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$

9. Find a basis for the null space of $\begin{bmatrix} 1 & 2 & -1 & 0 \\ -3 & -4 & 1 & -4 \\ 2 & 5 & -3 & -2 \end{bmatrix}$.

(a) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ -2 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

10. Let $W = \text{Span}\{1 + x + x^2, 2 + 3x + 3x^2 + x^3, 1 - x^3, x + x^2 + x^3\}$. Find the dimension of W.

(a) 2 (b) 1 (c) 3 (d) 4 (e) 5

11. Let $B = \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ be a basis for the vector space of 2×2 matrices and let $\mathbf{v} = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$. Find $[\mathbf{v}]_B$, the coordinates of $\mathbf{v}$ with respect to the basis B.

(a) $\begin{bmatrix} 2 \\ -1 \\ 3 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 3 \\ -1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ -1 \\ 5 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 5 \\ -1 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

12. Let A be a 3×3 matrix whose null space is spanned by $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Determine which of the following statements is true.

(a) $\text{rank}A = 2$ (b) $\det A \neq 0$ (c) A is invertible.

(d) $\text{rank}A = 1$ (e) A must be symmetric.

13. Suppose that y_1 and y_2 are solutions of $ty'' - y' + e^{t^2} y = 0$ and their Wronskian satisfies $W(y_1, y_2)(1) = 1$. Find $W(y_1, y_2)(t)$.

(a) t (b) e^{t^2} (c) $1/t$

(d) e^{1-t} (e) *cannot be determined*

14. Solve the initial value problem $2y'' - 12y' + 18y = 0, \quad y(0) = 1, \quad y'(0) = 2.$

(a) $y = (1-t)e^{3t}$ (b) $y = e^{3t}$ (c) $y = e^{3t} - t$

(d) $y = e^{-3t} + 5te^{-3t}$ (e) $y = e^t \cos(3t) + e^t \sin(3t)$

15. Find the general solution of $y'' - 4y' + 13y = 0.$

(a) $c_1 e^{2t} \cos(3t) + c_2 e^{2t} \sin(3t)$ (b) $c_1 e^{3t} \cos(2t) + c_2 e^{3t} \sin(2t)$ (c) $c_1 \cos(2t) + c_2 \sin(2t)$

(d) $c_1 e^{2t} + c_2 e^{-2t}$ (e) $c_1 e^{5t} + c_2 e^{-t}$

16. Find the solution to the initial value problem $y^2 y' + xy^3 + x = 0, \quad y(2) = 1.$

(a) $\left(2e^{6-3x^2/2} - 1\right)^{1/3}$ (b) $e^{-2+x^2/2}$ (c) $\left(2e^{6-3x^2/2}\right)^{1/3} - 1$

(d) $\left(e^{-3x^2/2} - 1\right)^{1/3}$ (e) $e^{2-x^2/2}$

17. Determine which of the following functions is an integrating factor for the equation $x\,dx + (x^2 + e^{y^2})\,dy = 0.$

(a) e^{2y} (b) $2y$ (c) e^{-2x} (d) $\dfrac{1}{x}$ (e) xe^{2y}

18. A tank initially contains 100 liters of pure water. A mixture containing a concentration of 2 grams per liter of salt enters the tank at a rate of 3 liters per minute, and the well-stirred mixture leaves the tank at the same rate. Find the amount of salt, in grams, in the tank after t minutes.

(a) $200 - 200e^{-0.03t}$ (b) $e^{6t} - 1$ (c) $6t$ (d) $100e^{-6t} - 100$ (e) $200e^{0.03t} - 200$

19. Find the form of a particular solution of $y'' + 2y' + y = e^{-t}(\sin(t) + t).$

(a) $y_p = e^{-t}\left(A\sin(t) + B\cos(t) + Ct^2 + Dt^3\right)$ (b) $y_p = t^2 e^{-t}\left(A\sin(t) + B\cos(t) + C + Dt\right)$

(c) $y_p = e^{-t}\left(A\sin(t) + B\cos(t) + Ct^3\right)$ (d) $y_p = t^2 e^{-t}\left(A\sin(t) + B\cos(t) + C\right)$

(e) $y_p = e^{-t}\left(A\sin(t) + B\cos(t) + C + Dt\right)$

20. Given that $y_1 = e^t$ and $y_2 = t$ are solutions of $(1-t)y'' + ty' - y = 0$, find a particular solution of $(1-t)y'' + ty' - y = e^t(1-t)^2$.

(a) $y_p = e^t\left(t - t^2/2\right)$ (b) $y_p = e^t(1-t)$ (c) $y_p = e^{2t} + te^t$

(d) $y_p = te^t + t^2$ (e) $y_p = e^t(1-t)^3/3$