MATH 228: Intro to Linear Alg and Diff Eqns Name:____________________________

Exam I *February 19, 2004* Instructor:____________________________

There are 12 problems on 7 pages worth a total of 90 points. You start with 10 points.

The first 8 problems are multiple choice questions worth 5 points each. Record your answers to these problems by placing an × through one letter for each problem on this page.

Problem 9 consists of 10 statements that you must mark true or false by placing an × through the letter T or F, respectively. Each correct answer in this problem is worth 2 points (20 points total).

Problems 10–12 are partial credit problems worth 10 points each. *You must show your work and all important steps to receive credit for these problems.*

Calculators are *not* allowed.

Compute the determinant of

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 3 & 4 & 5 & 0 \\ 6 & 0 & 7 & 8 \\ 9 & 0 & 10 & 11 \end{bmatrix}$$

-36

12

0

308

-128

Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 5 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 6 & 3 \end{bmatrix}$. Compute the (1,2)-entry of the matrix product AB.

4

-23

-1

2

not defined

Determine the angle between the vectors $(-1, 2, -1, 0)$ and $(1, -1, 3, -1)$.

$3\pi/4$

$\pi/2$

$2\pi/3$

$-\pi/6$

$-\pi/4$

Determine the matrix of the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by rotation by 90 degrees in the xy-plane, followed by reflection through the xz-plane and projection onto the yz-plane.

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation that satisfies $T(1,0) = (-1,2)$ and $T(0,1) = (3,-1)$. Find $T(4,5)$.

$(11,3)$

$(-4,10)$

$(12,-5)$

$(6,7)$

$(8,5)$

Let A be a 3×3 matrix with $\det(A) = 4$. Find $\det(2A^{-1})$.

2

1/2

1

1/4

4

Let $A = \begin{bmatrix} 1 & -2 & 0 \\ -3 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$. Compute the $(3,2)$-entry of A^{-1}.

$-4/3$

4

$-5/3$

2/3

2

Determine the dimension of the space of solutions to $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -6 & 3 \\ 2 & 4 & -2 \end{bmatrix}$.

2

1

3

0

4

9. Mark each statement true or false by placing an $\times$ through the letter T or F, respectively.

T F The inverse of an upper-triangular matrix is an upper-triangular matrix.

T F The matrix product of two symmetric matrices is a symmetric matrix.

T F A matrix A is invertible if and only if $\det(A) \neq 0$.

T F A linear system $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ if and only if $\det(A) = 0$.

T F A linear transformation $T : \mathbf{R}^n \to \mathbf{R}^n$ is one-to-one if and only if the range of T is $\mathbf{R}^n$.

T F If $A\mathbf{x} = \mathbf{0}$ for some $\mathbf{x} \neq \mathbf{0}$, then $\mathbf{x}$ is an eigenvector of A.

T F If A is an $n \times n$ matrix and $\mathbf{u}$, $\mathbf{v}$ are vectors in $\mathbf{R}^n$, then $A\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot A\mathbf{v}$.

T F The set of matrices of the form $\begin{bmatrix} 1+x & y \\ y & 1-x \end{bmatrix}$ is a vector subspace of the vector space of 2×2 matrices.

T F The set of functions $f(x)$ such that $f'(2) = f(1)$ is a vector subspace of $C^1[0,3]$.

T F If $\text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\} = \mathbf{R}^n$ then $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is a basis for $\mathbf{R}^n$.

10. Determine the solution of the linear system

$$
\begin{array}{ccccccccccc}
x_1 & + & x_2 & & & + & x_4 & + & x_5 & = & 2 \\
x_1 & & & + & 2x_3 & & & + & x_5 & = & 2 \\
& & x_2 & + & x_3 & + & x_4 & & & = & 3 \\
x_1 & & & + & x_3 & & & + & x_5 & = & 1 \\
x_1 & + & x_2 & + & 2x_3 & & x_4 & + & x_5 & = & 4
\end{array}
$$

11. Compute the eigenvalues and eigenvectors of the linear transformation $T(x, y) = (2x + 2y, -x + 5y)$.

12. Let P_3 be the vector space of polynomials of degree ≤ 3. Show that the set of vectors in P_3,

$$\mathbf{v}_1 = 1 + x + x^2, \quad \mathbf{v}_2 = x + x^2 + x^3, \quad \mathbf{v}_3 = x + x^3, \quad \mathbf{v}_4 = 1 + 3x + x^2 + 2x^3,$$

is linearly dependent.