MATH 228C: Intro to Linear Alg and Diff Eqns Name:______________________________

Final Exam *May 6, 2004*

There are 20 multiple choice questions worth 7 points each for a total of 140 points. You start with 10 points. Record your answers by placing an × through one letter for each problem on this page.

Calculators are *not* allowed.

Find the general solution of the linear system

$$\begin{array}{rcrcrcrcl} x_1 & - & x_2 & + & 2x_3 & + & x_4 & = & 1 \\ & & x_2 & + & x_3 & - & x_4 & = & 0 \\ x_1 & & & + & x_3 & + & 2x_4 & = & 3 \end{array}$$

$x_1 = 4 - 3t,\ x_2 = 1,\ x_3 = -1 + t,\ x_4 = t$

$x_1 = 4,\ x_2 = 1,\ x_3 = -1,\ x_4 = 0$

$x_1 = 4 - t,\ x_2 = 1,\ x_3 = -1 - t,\ x_4 = t$

$x_1 = 1 + s - 3t,\ x_2 = s,\ x_3 = -1 + t,\ x_4 = t$

$x_1 = s - t,\ x_2 = -1 + s,\ x_3 = -1 - t,\ x_4 = t$

Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$. Compute the matrix product $A^{-1}B$.

$\begin{bmatrix} 3 & 2 & -1 \\ -4 & -2 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 6 & 13 \\ 0 & -2 & 5 \end{bmatrix}$

$\begin{bmatrix} -1 & 2 & 4 \\ -3 & -1 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 & -1 \\ -4 & 2 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 13 & 6 \\ 0 & 5 & -2 \end{bmatrix}$

Determine which of the following is an inner product on the vector space of 2×2 matrices.

$\langle A, B \rangle = \text{tr}(A^T B)$

$\langle A, B \rangle = \det(AB)$

$\langle A, B \rangle = \det(A + B)$

$\langle A, B \rangle = \text{tr}(A + B)$

$\langle A, B \rangle = A^T B$

Compute the determinant of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix}$.

-28

24

-18

0

12

Find a basis for $\text{span}\{(1,-1,1,0),(-1,1,0,1),(1,-1,2,1)\}$.

$\{(1,-1,1,0),(0,0,1,1)\}$

$\{(1,-1,1,0),(-1,0,1,1)\}$

$\{(1,-1,1,0),(0,0,2,1),(0,0,0,1)\}$

$\{(1,-1,1,0)\}$

$\{(1,0,0,0),(0,1,0,0),(0,0,1,0)\}$

Let V be the vector subspace of continuous functions with basis $B=\{\cos(x),\sin(x),x\cos(x),x\sin(x)\}$. Find the matrix of the linear transformation $T:V\to V$ defined by $T(f(x))=f'(x)$ with respect to B.

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

The Gram-Schmidt process is applied to the following basis of $\mathbf{R}^3$: $u_1=(2,2,1)$, $u_2=(-2,1,2)$, $u_3=(0,0,2)$, resulting in the orthonormal vectors $q_1=(2/3,2/3,1/3)$, $q_2=(-2/3,1/3,2/3)$, and q_3. Find q_3.

$(1/3,-2/3,2/3)$

$(1/3,2/3,-2/3)$

$(2/3,-1/3,-2/3)$

$(0,0,1)$

$(2/3,2/3,1/3)$

Find the line $y=ax+b$ that is the best least-squares fit through the points $(1,1)$, $(1,2)$ and $(2,3)$.

$y=\frac{3}{2}x$

$y=\frac{5}{4}x+\frac{1}{8}$

$y=\frac{5}{2}x-\frac{1}{4}$

$y=\frac{4}{3}x+\frac{1}{8}$

$y = x + \frac{1}{2}$

A matrix A has eigenvectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ with corresponding eigenvalues 1, 2, and 3. Use this information to reconstruct the matrix A.

$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

Suppose $B = \{v_1, v_2, v_3\}$ and $C = \{u_1, u_2, u_3\}$ are bases of a vector space V related by $v_1 = u_1 - u_3$, $v_2 = u_2 - u_3$, $v_3 = u_3 - u_2$. If $[x]_B = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, find $[x]_C$.

$\begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

Suppose $y(t)$ satisfies the initial value problem $y' - \tan(t)y = \sec(t)$, $y(0) = 0$. Find $y(\pi/4)$.

$\pi\sqrt{2}/4$

$\pi/4$

$\pi\sqrt{2}/8$

$\pi/8$

0

Find an integrating factor for $(x^3 + xy^3 + y^3)\,dx + 3xy^2\,dy = 0$.

e^x

$3xy^2$

e^y

$3x^2y$

x

Suppose the air resistance on a high-speed 1 kg projectile moving in a straight line is proportional to the square of its speed. The projectile has an initial speed of 500 meters per second (mps) and the air resistance at that speed is 25 newtons. Find the speed of the projectile after 5 seconds. (Ignore other forces.)

400 mps

250 mps

300 mps

350 mps

200 mps

Solve the initial value problem $y'' - 2y' + y = 0$, $y(1) = 0$, $y'(1) = 1$.

$(t-1)e^{t-1}$

te^t

$e^t - te^t$

$(t-1)e^t/2$

$(te^{t-1} - 1)/2$

Find the general solution of $y'' - 2y' - 3y = 0$.

$c_1e^{-t} + c_2e^{3t}$

$c_1e^{t} + c_2e^{-3t}$

$c_1e^{-2t} + c_2e^{-3t}$

$c_1e^{2t} + c_2e^{3t}$

$e^{-t}(c_1 + c_2t)$

Compute the Wronskian of $y_1 = t\sin(t)$ and $y_2 = t\cos(t)$.

$-t^2$

$-2t\sin(t)\cos(t)$

$-t^2\sin(t)\cos(t)$

$-t^2(\sin^2(t) - \cos^2(t))$

-1

Let L be the linear operator $L[y] = y'' + p(t)y' + q(t)y$. Given that $\{t+1, e^t\}$ is a basis for the kernel of L, find a function $\phi(t)$ that satisfies $L[\phi(t)] = te^t$.

$t^2e^t/2$

$t^2/2 + t - e^t$

$(t^2/2 + t - 1)e^t$

$(t-1)e^t$

$(t^2-1)e^t$

Find a particular solution to $y'' + y' + y = t^3$.

$t^3 - 3t^2 + 6$

$e^{-t/2}(\cos(\sqrt{3}t) + \sin(\sqrt{3}t)) + t^3/6$

$e^{-t/2}(\cos(\sqrt{3}t) - \sin(\sqrt{3}t))/\sqrt{3} + t^3 - 3t^2$

$e^{-t/2}(2\cos(\sqrt{3}t) + \sin(\sqrt{3}t)/\sqrt{3})$

$(3t^2 - t^3)/6$

Suppose a spring-mass system is governed by the differential equation $2u'' + 4u' + 3u = 0$ with initial conditions $u(0) = 1$, $u'(0) = 0$. Determine which of the following adjectives best describes the system.

oscillating

overdamped

undamped

critically damped

resonating

Determine which of the following is a *stable* equilibrium of the autonomous system $\frac{dy}{dt} = 2y - 3y^2 + y^3$.

1

2

3

4

0