Name: ________________________________ **Instructor:** Jeffrey Diller

Math 230: Ordinary Differential Equations
Spring Semester 1999
Exam 1
Thursday, February 18

This Examination contains 9 problems on 7 sheets of paper including the front cover. Do all your work on the paper provided and show your computations.

Scores

Question	Possible	Actual
1	10	
2	10	
3	10	
4	5	
5	15	
6	10	
7	10	
8	15	
9	20	
Total	100	

GOOD LUCK

1. Solve the initial value problem

$$ty' - 3y = (t+1), \qquad y(1) = 2.$$

(10 points)

2. Find a canonical set of solutions for

$$y'' + 7y' - 8y = 0$$

at the initial point $t_0 = 0$. (10 points)

3. Consider the initial value problem

$$y' = 2e^{3yt}, \qquad y(0) = 1.$$

(a) Use Euler's method with a single step to approximate $y(.1)$. (8 points)

(b) Estimate the difference between your approximation and the actual value of $y(.1)$ (7 points)

4. Suppose that I use the Runge-Kutta method with a step size of $h = .25$ to approximate $y(1) \approx 1.3345$, where $y(t)$ is the solution of an initial value problem

$$y' = f(y, t), \quad y(0) = 1.$$

Suppose I believe that this approximation is accurate to 3 decimal places. About what value should I take for h if I want my approximation of $y(1)$ to be accurate to 11 decimal places? (5 points)

5. Consider (but **do not try to solve**) the differential equation

$$y' = y^4 - 2y^3.$$

(a) Find and classify all equilibrium solutions. (8 points)

(b) On a single plot, sketch the graphs of the solutions y_1, y_2, y_3 satisfying $y_1(0) = 1$, $y_2(0) = 2$, $y_3(0) = 3$. Make sure label to label the graphs! (7 points)

6. Note that $y_1(t) = e^t$, $y_2(t) = e^{t-1}$ are both solutions of

$$y'' - y' = 0.$$

Verify that $\{y_1, y_2\}$ is not a fundamental set of solutions by evaluating the Wronskian of y_1 and y_2 at $t = 0$. Then give initial values y_0, y_0' such that no linear combination $y = C_1 y_1 + C_2 y_2$ satisfies $y(0) = y_0$ and $y'(0) = y_0'$. (10 points)

7. Consider (but **do not try to solve**) the initial value problem

$$y' = \left(y + \frac{1}{t}\right)^{2/3}, \qquad y(t_0) = y_0.$$

For what initial values t_0 and y_0 do the hypotheses of the existence and uniqueness theorem for first order non-linear ODE's fail? (10 points)

8. Succumbing to Detroit's relentless marketing, I give into my irrational desire to have a sport utility vehicle. I rush to the nearest Lincoln dealer and inquire about prices for their Navigator SUV. The sales manager tells me that they'll sell me this vehicle for \$45,000 cash. Noticing the crestfallen look on my face, he puts his arm around my shoulder and tells me that though he wouldn't normally do this, he'll let me drive the Navigator away for free—they'll just loan me the money at 10% continuously compounded interest, and I can pay the vehicle off in continuous payments over five years. Heck, he'll even knock the asking price down to \$40,000. At this point my disappointment turns to anger, I fling off his unwanted and insincere arm and storm out of the dealership resolved to endure my Dodge Neon a few years longer. How much would I have paid if I had accepted the dealer's second offer? (15 points)

9. Below are direction fields for four first order differential equations. Identify the direction field corresponding to

(a) a linear homogeneous equation;

(b) an autonomous equation;

(c) an equation of the form $\dfrac{dy}{dt} = f(t)$;

(d) a non-linear and non-autonomous equation;

(5 points each)

(i) (ii)

(iii) (iv)