Name: ________________________________ **Instructor:** Jeffrey Diller

Math 230: Ordinary Differential Equations
Spring Semester 1999
Exam 2
Thursday, April 15

This Examination contains 7 problems on 8 sheets of paper including the front cover. Do all your work on the paper provided and show your computations.

Scores

Question	Possible	Actual
1	25	
2	15	
3	15	
4	10	
5	10	
6	10	
7	15	
Total	100	

GOOD LUCK

1. Consider the inhomogeneous differential equation.

$$y'' + 4y' + 4y = e^{2t} + e^{-2t}$$

(a) Find the general solution of the corresponding *homogeneous* equation. (10 points)

(b) Find the general solution of the (inhomogeneous) equation. (10 points)

(c) Find an equivalent first order system of differential equations, and express this system using matrix/vector notation. (5 points)

2. Consider the system

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

(a) Present the solution in real form (i.e. without imaginary numbers). (10 points)

(b) Find the particular solution satisfying

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(5 points)

3. Let $\sum_{n=0}^{\infty} c_n x^n$ be a power series solving

$$y'' + (x+1)y = 0, \qquad y(0) = 1, y'(0) = 0.$$

Find a recursion formula for the coefficients c_n and give the first four non-vanishing terms in the series. (15 points)

4. Consider the functions $e^x, x+1$ on the interval $(-\frac{1}{2}, \frac{1}{2})$.

(a) Are these functions linearly independent? Why or why not? (5 points)

(b) Could there be a differential equation $y''(x)+p(x)y'(x)+q(x) = 0$ having both these functions as solutions? Why or Why not? Assume $p(x)$ and $q(x)$ are continuous for $x \in (-\frac{1}{2}, \frac{1}{2})$. (5 points)

5. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ solves the initial value problem

$$y'' + \frac{1}{x^2+2}y = 0, \qquad y(0) = 0, y'(0) = 1.$$

(a) What is c_3? (Hint: do **not** try to find a recursion formula for the coefficients c_n.) (5 points)

(b) What is the (smallest possible) radius of convergence of the series? (5 points)

6. Find and classify the singular points of

$$(e^x - 1)(x-1)^2 y'' + xy' + \frac{x-1}{x} y = 0.$$

(10 points)

7. Each of the following three pictures represents a direction field or phase portrait for a 2×2 linear, homogeneous, constant coefficient system of ODE's—that is, a system of the form

$$\mathbf{x}'(t) = A\mathbf{x}(t),$$

where $\mathbf{x}(\mathbf{t}) = (x_1(t), x_2(t))$ is a vector-valued function and A is 2×2 constant matrix. Note that A is different from one picture to the next.

For each picture, say what you can about the eigenvalues and eigenvectors of the matrix A in the corresponding system. (5 points each)

(a)

(b)

(c)