Math 230: Some Practice Problems for the First Exam
Fall Semester 1998

1. Solve the following initial value problem and determine the interval where the solution is defined.

$$(t^2-9)\frac{dy}{dt}+2ty=\frac{2t^2-18}{t^2}, \quad y(1)=-1.$$

2. Circle the differential equation whose direction field is shown in the following picture.

A. $y'=y(x-y)$ **B.** $y'=(x-y)(x+y)$ **C.** $y'=(1-y)(x-y)$

D. $y'=(y-1)(x-y)$ **E.** $y'=(x-1)(x-y)$

3. A tank initially contains 300 gallons of pure water. A mixture containing 1 lb of salt per gallon enters the tank at a rate of 3 gal/min. The well-stirred mixture leaves the tank at a rate of 5 gal/min.
Write the initial value problem needed to find the amount of salt $S(t)$ in the tank at time $t>0$ prior to the instant when the tank is empty. **Do not solve it!**
[Hint: First you should figure out how many gallons of mixture are in the tank at time t.]

4. Given the differential equation

$$\frac{dy}{dt}=2y(y-1)(y-3).$$

Do not attempt to solve it!

(a) Find the (constant) equilibrium solutions and classify each one as asymptotically stable, unstable, or semistable.

(b) Sketch the graphs of the four solutions y_1, y_2, y_3, and y_4 which have initial condition $y_1(0)=1$, $y_2(0)=2$, $y_3(0)=3$, and $y_4(0)=0.5$. Neglect concavity.

5. Find the solution of the initial value problem

$$6y''-7y'+y=0, \quad y(0)=0, \quad y'(0)=5.$$

6. Determine the largest interval in which the following initial value problem is certain to have a unique twice differentiable solution.

$$(t-3)(t-1)ty''+t^2y'+(t-1)y=\sin t, \quad y(2)=-3, \quad y'(2)=4.$$

Do not attempt to find the solution!

7. Use Euler's method with a step size of .01 to estimate $y(2.01)$ where y solves the initial value problem

$$y'+\cos(\pi ty)=0.$$

Estimate the difference between the approximate and actual values of y at $t=2.01$.

8. The textbook is a good source of other problems to work. Note in particular the miscellaneous problems on page 94. These will be a good source of first order ODE problems. Be careful, though–we didn't cover techniques for solving non-linear ODE's that aren't separable, so ignore any problem involving these kinds of ODE's.