Name: ______________________________ **Instructor:** Jeffrey Diller

Math 230: Ordinary Differential Equations
Spring Semester 2000
Exam 2
Friday, April 14

This Examination contains 5 problems on 7 sheets of paper including the front cover. Do all your work on the paper provided and show your computations.

Scores

Question	Possible	Actual
1	25	
2	15	
3	15	
4	40	
5	20	
Total	100	

GOOD LUCK

1. Consider the inhomogeneous differential equation

$$y'' + 4y = 2e^t$$

(a) Find the solution of the corresponding *homogeneous* equation that satisfies $y(\pi) = 2$ and $y'(\pi) = 1$. (10 points)

(b) Find the general solution of the (inhomogeneous) equation. (10 points)

(c) Find an equivalent first order system of differential equations. (5 points)

2. Consider the inhomogeneous differential equation

$$y''' + 2y'' + y' = t + e^{2t}\cos t.$$

Write down the correct form of the general solution to this equation *without evaluating any of the unknown constants*. Indicate which part will be the particular solution and which is the solution to the homogeneous equation. (15 points)

3. Consider the system

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 4 & -1 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

(a) Find the general solution. (10 points)

(b) Find the particular solution satisfying

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(5 points)

4. Suppose that $\mathbf{x}' = A\mathbf{x}$ is a 2×2 system of ODE's, and that A is a constant matrix. Suppose that

$$\mathbf{x}^1(t) = \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{-2t} \qquad \text{and} \qquad \mathbf{x}^2(t) = \begin{pmatrix} -2 \\ 4 \end{pmatrix} e^{3t}$$

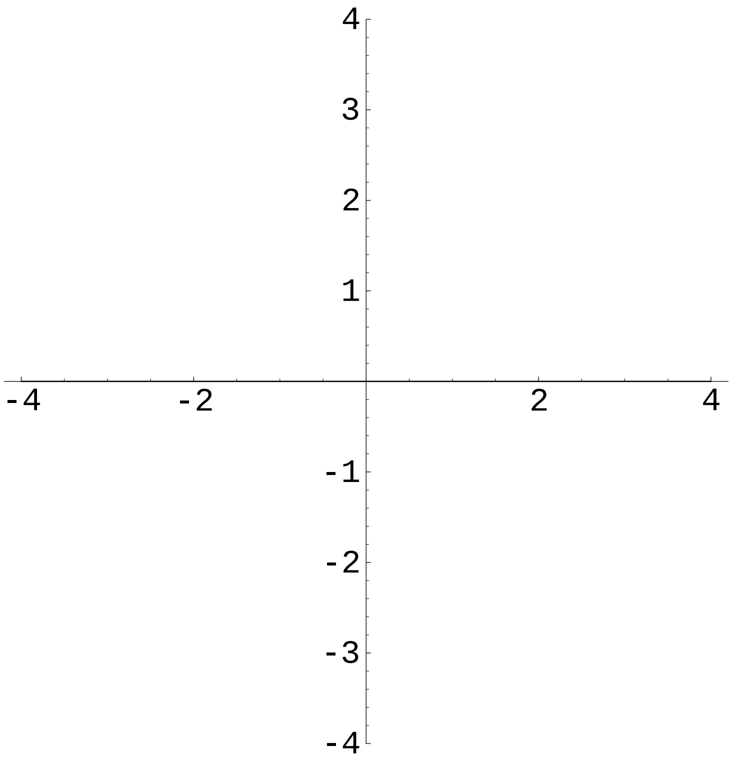
are solutions to this system.

(a) Write down the general solution of the system, and explain briefly why your answer gives *all* the solutions to the system. (8 points)

(b) What are the eigenvalues and eigenvectors of A? What is e^{At}? You don't need to simplify your answer to the second question. (10 points)

(c) What sort of critical point is $(0,0)$ for this system? Is it stable or unstable? (5 points)

(d) On the graph below, plot solution curves for solutions $\mathbf{x(t)}$ satisfying $\mathbf{x}(0) = (-1, 1)$ and $\mathbf{x}(0) = (0, -1)$. Draw arrows on your solution curves to indicate the direction for increasing t. (7 points)



(e) Suppose that we change the system to

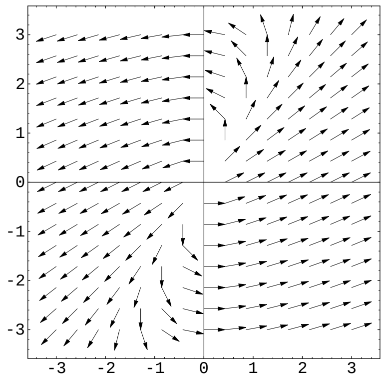
$$\mathbf{x}' = A\mathbf{x} + \begin{pmatrix} t \\ \sin t \end{pmatrix}$$

where A is the same matrix as before. Write down the solution to this new system. *Do* **not** *simplify your answer (i.e. don't perform any matrix multiplication, inversion, integration, differentiation, conflagration, indentation, inflation, etc.)* (10 points)

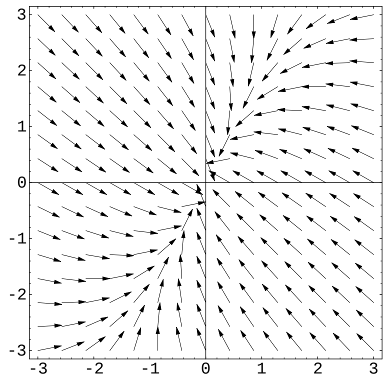
5. Each of the following three pictures represents a direction field or phase portrait for a 2×2 linear, homogeneous, constant coefficient system of ODE's—that is, a system of the form

$$\mathbf{x}'(t) = A\mathbf{x}(t),$$

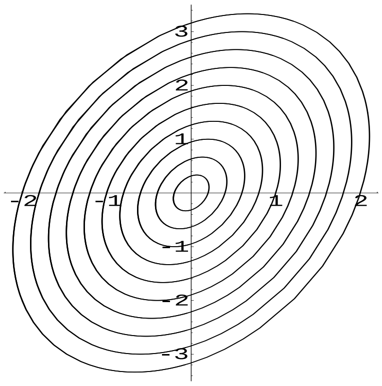
where $\mathbf{x}(\mathbf{t}) = (x_1(t), x_2(t))$ is a vector-valued function and A is 2×2 constant matrix. For each picture, say what you can about the eigenvalues and eigenvectors of the matrix A in the corresponding system. Also, say whether the origin is a stable or unstable critical point. (20 points total)



(a)



(b)



(c)