Math 230, Spring 1999
Mathematica assignment 1
Sample Solutions

The following problems are taken from pages 31 to 34 of the *Differential Equations with Mathematica* book. These are strictly optional, but I recommend going through at least some of them in order to refresh your memory of *Mathematica* syntax.

Problem 1 (b and d only) :

b.

```
2^123
```

```
10633823966279326983230456482242756608
```

c.

```
N@Pi, 10D
N@22  7, 10D
N@311  99, 10D
N@355  113, 10D
```

```
3.141592654

3.142857143

3.141414141

3.14159292
```

355/113 is the best approximation of p,and 22/7 is the worst.

Problem 2:

a.

```
N@Sin@1D  .1, 10D
```

```
0.9983341665
```

b.

```
N@Sin@.01D  .01, 10D
```

```
0.9999833334
```

c.

```
N@Sin@.001D  .001, 10D
```

```
0.9999998333
```

```
9
```

Problem 3:

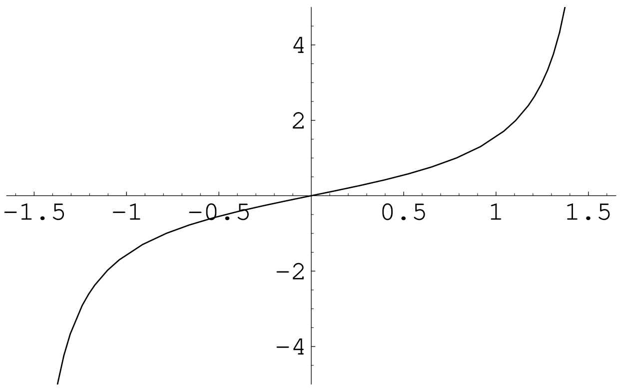
```
Factor@x^3 - y^3D
```

```
Hx - yL Hx^2 + x y + y^2L
```

Problem 4 (d and e):

d.

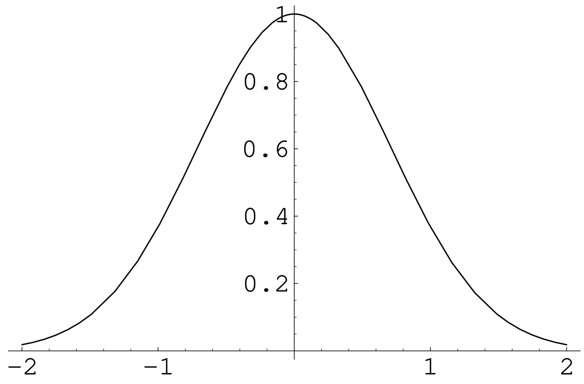
```
Plot@Tan@xD, 8x, -p  2, p  2<, PlotRange -> 8-5, 5<D
```



(Notice that I used the PlotRange Option to keep the scale on the vertical axis reasonable.)

e.

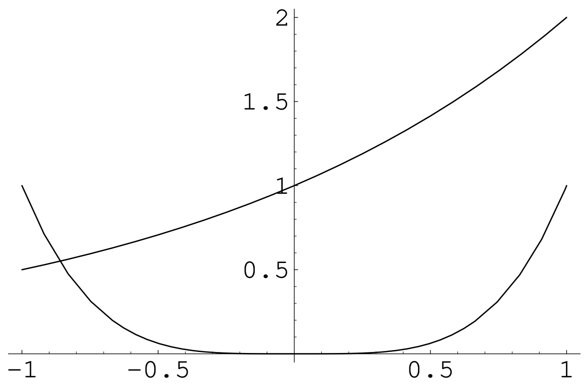
```
Plot@E^H- x^2L, 8x, - 2, 2<D
```



Problem 5:

I'm going to record all my attempts here. Fiddling around a bit is part of the game...

```
Plot@8x^4, 2^x<, 8x, - 1, 1<D
```

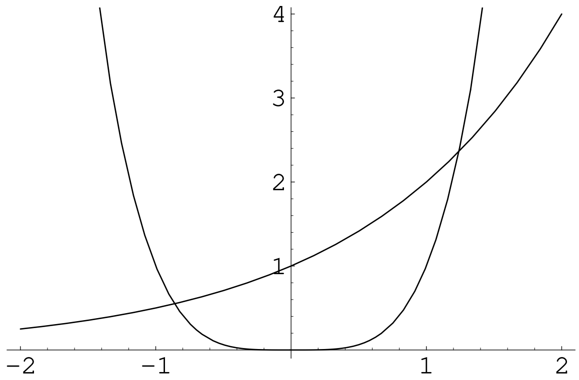


OK, looks like we've got a crossing at about x = –.9.

```
FindRoot@x^4 - 2^x == 0, 8x, - .9<D
```

```
8x fi -0.861345<
```

```
Plot@8x^4, 2^x<, 8x, -2, 2<D
```



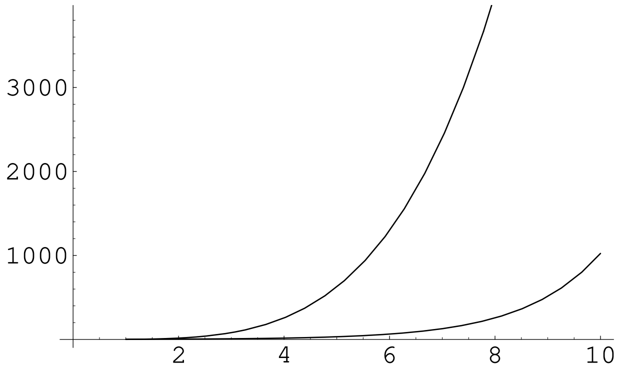
We've got another crossing at about 1.3

```
FindRoot@x^4 - 2^x == 0, 8x, 1.3<D
```

```
8x fi 1.23963<
```

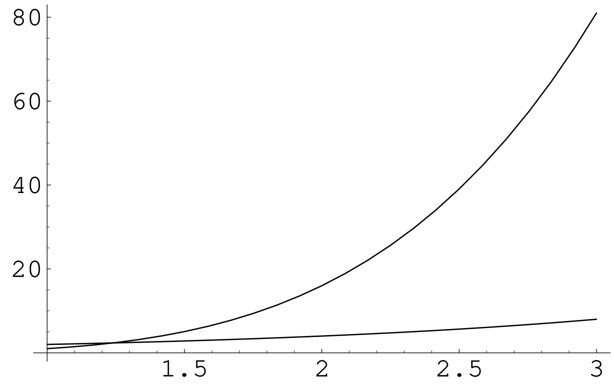
Now x^4 goes to infinity as x goes to negative infinity, but 2^x goes to zero -- not likely that we'll find any more crossings for negative values of x. On the other hand 2^x grows much more quickly than x^4 as x goes to infinity. There must be at least one more positive value of x at which the graphs cross.

```
Plot@8x^4, 2^x<, 8x, 1, 10<D
```



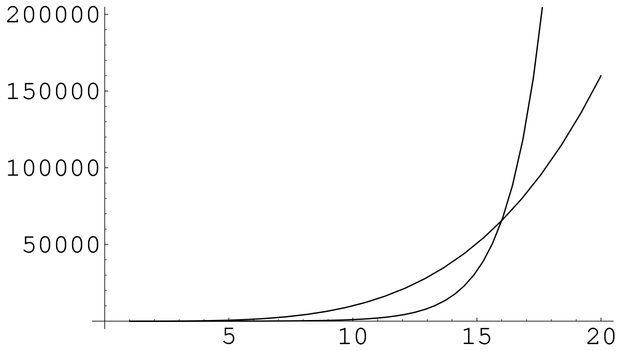
Didn't learn much from this one! Try again...

```
Plot@8x^4, 2^x<, 8x, 1, 3<D
```



Maybe I didn't try for a large enough x:

```
Plot@8x^4, 2^x<, 8x, 1, 20<D
```



Aha! Looks like there's a crossing near x = 16.

```
FindRoot@82^x == x^4<, 8x, 16<D
```

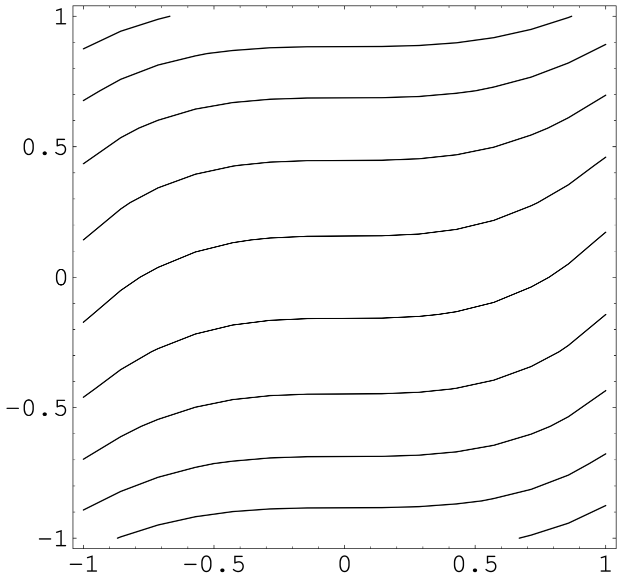
```
8x fi 16.<
```

So x=16 was a pretty good guess! All told, we found crossings at x = −.861,x= 1.23, and x =16.

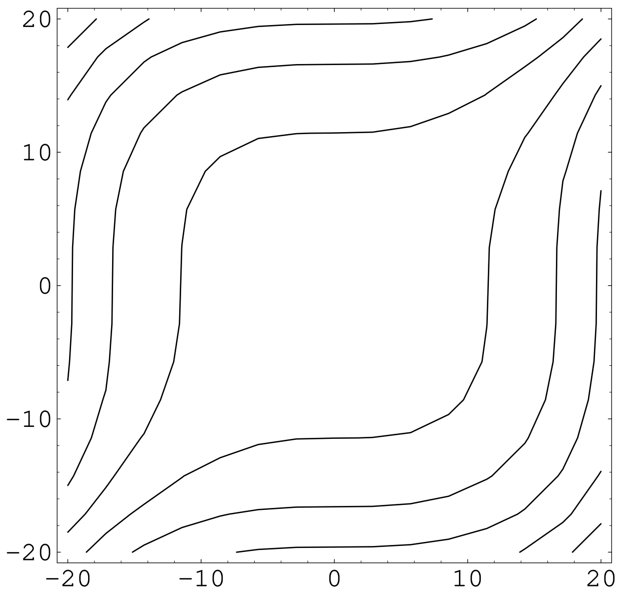
Problem 6 (a and c):

a.

```
ContourPlot@3 y + y^3 - x^3, 8x, - 1, 1<, 8y, - 1, 1<, ContourShading -> FalseD
```



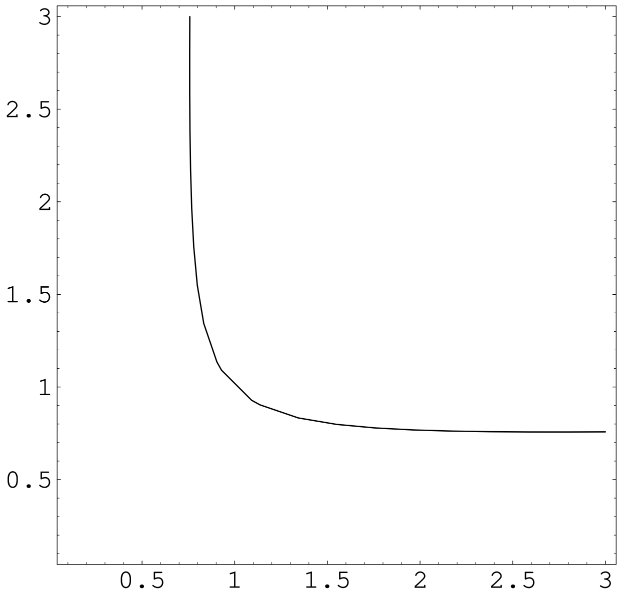
```
ContourPlot@3 y + y^3 - x^3, 8x, - 20, 20<, 8y, - 20, 20<, ContourShading -> FalseD
```



c.

Since f(1,1) = 0, we only want the level curve f(x,y) = 0. We need to be careful with the ranges for x and y, since the natural log function
isn't defined at zero or at negative numbers.

```
ContourPlot@y Log@xD + x Log@yD, 8x, .1, 3<,
 8y, .1, 3<, Contours -> 80<, ContourShading -> FalseD
```



Problem 7 (c only):

```
D@ArcSin@2 x + 3D, xD
```

```
        2
" ##################
  1 - H3 + 2 xL
```

Problem 8:

a.

```
polly = x^6 - 21 x^5 + 175 x^4 - 735 x^3 + 1624 x^2 - 1764 x + 720
```

$720 - 1764\,x + 1624\,x^2 - 735\,x^3 + 175\,x^4 - 21\,x^5 + x^6$

```
Factor@pollyD
```

H− 6 + xL H−5 + xL H−4 + xL H−3 + xL H−2 + xL H−1 + xL

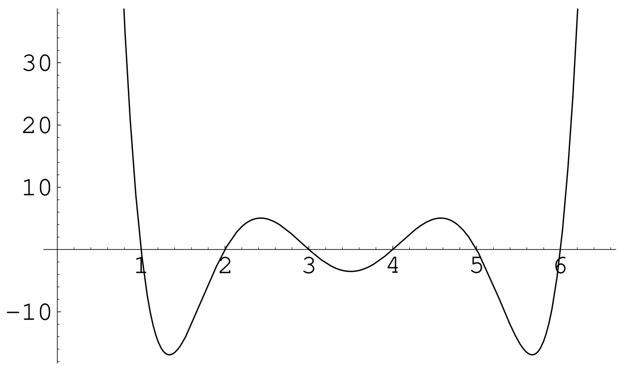
b.

```
Solve@polly == 0, xD
```

88x fi 1<, 8x fi 2<, 8x fi 3<, 8x fi 4<, 8x fi 5<, 8x fi 6<<

c.

```
Plot@polly, 8x, .5, 6.5<D
```

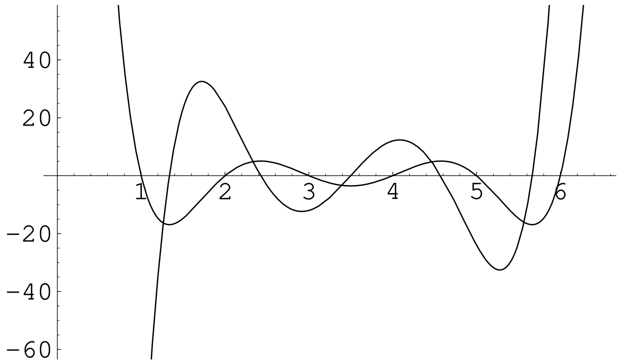


d.

```
dpolly = D@polly, xD
```

$-1764 + 3248\,x - 2205\,x^2 + 700\,x^3 - 105\,x^4 + 6\,x^5$

```
Plot@8polly, dpolly<, 8x, .5, 6.5<D
```



Problem 9 (a and d):

a.

```
Limit@Sin@xD  x, x -> 0D
```

```
1
```

d.

```
Limit@1  Hx - 1L, x -> 1, Direction -> 1D
```

```
-¥
```

Problem 10 (a and g):

a.

```
Integrate@Sin@xD, 8x, 0, Pi  2<D
```

```
1
```

g.

```
Integrate@Exp@Cos@xDD, xD
```

```
 E^Cos@xD  x
```

Looks like not even Mathematica can handle this one. Nonetheless, we can differentiate the unevaluated integral to see whether we get back the original function:

```
D@%, xD
```

```
E^Cos@xD
```

Problem 11 (b only)

```
NIntegrate@Sqrt@x^4 + 1D, 8x, 0, 1<D
```

```
1.08943
```